Week In Review (week 6)

Math 151 - Fall 2022
Liwei Chen

Problems:

1. Differentiate the following functions.
   (a) $f(x) = x^7 + \sqrt[4]{x} - \frac{5}{x} + \tan(x) - \csc(x)$
   (b) $h(t) = (2t+5)(3-t)$
   (c) $y = \left(\frac{1}{x^2} - \frac{7}{x^5}\right)(3x-2)$
   (d) $y = \frac{\sqrt[4]{x^3}+x}{x^2}$
   (e) $p(t) = (2-3t+5t^2)^{50}$
   (f) $f(x) = \frac{x^2e^x}{x^2+e^x}$
   (g) $H(t) = \frac{t}{(t^3-7)^5}$
   (h) $f(x) = 3^{5x^2-1}$
   (i) $k(t) = e^{t\sin^2(t)}$
   (j) $y = \csc(\tan(\cos(x)))$
2. Find the 2022nd derivative of $y = xe^{2x}$
3. Find the tangent line equation to the graph of $f(x) = 3x + \sqrt{x}$ at $(4, f(4))$.
4. Given $f(x) = \sqrt[3]{xg(x)+2}$. Let $g(2) = 3$ and $g'(2) = 1$. Find the normal line equation to the graph of $f(x)$ at $(2, f(2))$.
5. Given
$$f(x) = \begin{cases} mx - b & \text{if } x < -1 \\ 5x^2 & \text{if } x \geq -1 \end{cases}.$$
   Find values of m and b that make $f(x)$ differentiable everywhere.
6. For what values of a and b is the line $y = 3x - b$ tangent to the parabola $y = ax^2$ when $x = 4$?
7. Show there are two tangent lines to the parabola $y = 4x^2$ that pass through the point $(0, -4)$. Find the equation of these tangent lines.