Problems:

1. Sketch the graph of the following functions. Explain why the function is discontinuous at the given point a.

   (a) $f(x) = \frac{1}{x+2}$ and $a = -2$

   (b)
   $$f(x) = \begin{cases} x+3 & \text{if } x \leq -1 \\ 2^x & \text{if } x > -1 \end{cases} \quad \text{and} \quad a = -1$$

   (c)
   $$f(x) = \begin{cases} \cos x & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 - x^2 & \text{if } x > 0 \end{cases} \quad \text{and} \quad a = 0$$

2. Find the limits.

   (a)
   $$\lim_{x \to 4} 3^{\sqrt{x^2 - 2x - 4}}$$

   (b)
   $$\lim_{x \to \pi} \sin(x + \sin x)$$

3. Find the values of a and b that make f continuous everywhere.
   $$f(x) = \begin{cases} \frac{x^2 - 1}{x - 1} & \text{if } x < 1 \\ ax^2 + bx - 5 & \text{if } 1 \leq x < 2 \\ 3x - a + 2b & \text{if } x \geq 2 \end{cases}$$

4. Show that the equation $e^x = 3 - 2x$ has a root in the interval $(0, 1)$.

5. Find the limits.

   (a)
   $$\lim_{x \to \infty} \frac{x + 1}{4x - 3}$$

   (b)
   $$\lim_{x \to -\infty} \frac{x^2 - 1}{2x + 5}$$

   (c)
   $$\lim_{x \to -\infty} \frac{\sqrt{2x^2 + 3x + 1}}{x - 1}$$

   (d)
   $$\lim_{x \to \infty} \frac{1 + e^x}{1 - 3e^x}$$

   (e)
   $$\lim_{x \to \infty} \left[\ln(3x^2 + 4) - \ln(6x^2 - 5) \right]$$

   (f)
   $$\lim_{x \to -\infty} \left(\sqrt{x^2 + x + 1} + x \right)$$

6. Find the horizontal and vertical asymptotes of the function $f(x) = \frac{2e^x}{e^x - 5}$.

7. Find the equation of the tangent line to the graph of the function $f(x) = \sqrt{x}$ at $(1, f(1))$.

8. Find the equation of the tangent line to the graph of $y = g(x)$ at $x = 5$, given $g(5) = -3$ and $g'(5) = 4$.

9. The position function of a moving particle is given by $s(t) = 2t^2 - 6t + 5$, where t is the time.

   (a) Find the average speed of the particle over the interval $[4, 6]$.

   (b) Find the instantaneous velocity at $t = 4$.

10. The following limits represent the derivative of some function f at some point a. Find such an f and a.

   (a)

$$\lim_{h \to 0} \frac{\sin\left(\frac{\pi}{6} + h\right) - \frac{1}{2}}{h}$$

   (b)

$$\lim_{x \to 1/4} \frac{\frac{1}{x} - 4}{x - \frac{1}{4}}$$