Week in Review 13

Monday, December 5, 2022 5:02 PM

Q1. $f'(3)=0$ and $f''(3)=7$.

2^{nd}-derivative test: $x=3$ critical pt. ($f'(3)=0$).

$f''(3)=7>0$ " $\cup$ " $_{x=3}$

So f has a loc. min at $x=3$.

Choose (c).

Q2. $y=\frac{24}{x^2}+12x+b$

$y'=-\frac{48}{x^3}+12$ Set $-\frac{48}{x^3}+12=0$

$\Rightarrow 4=x^3 \Rightarrow x=\sqrt[3]{4}$.

critical pts: $x=\sqrt[3]{4}$ ($x=0$ is not defined).

$$\underset{\sqrt[3]{4}}{\overset{-\quad\;\; +}{\longrightarrow}}$$ sign of y'

y has a loc. min at $x=\sqrt[3]{4}$.

Choose (a).

Q3. f is NOT defined at $x=8$.

Critical pts: set $\frac{-7(x-1)(x+4)}{(x-8)^4}=0 \Rightarrow x=1$ and $x=-4$.

$$\overset{-\quad\;\; +\quad\;\; -}{\longrightarrow} \quad (-4,\ 1)$$ sign of f'

-4: loc. min, 1: loc. max

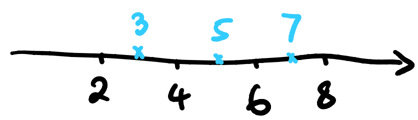
Choose (c).

Q4. $\lim_{x\to 0} \dfrac{e^{4x}-5-4x}{x^2} = \text{"} \dfrac{e^{0}-5-4\cdot 0}{0^2} \text{"} = \text{"} \dfrac{-4}{0^2} \text{"} = -\infty$

Choose (e).

Q5. $f(x) = x^2 - 1$. on $[2,8]$. 3 rectangles & midpoints.

$$\Delta x = \frac{8-2}{3} = 2$$



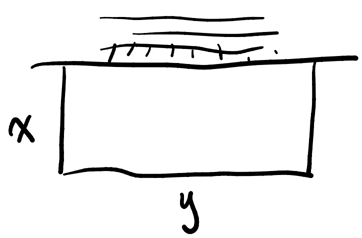
$$M_3 = [f(3) + f(5) + f(7)]\Delta x$$

$$= (8 + 24 + 48)\cdot 2$$

$$= 160$$

Choose (e).

Q6.



$2x + y = 1200$

$A = xy$. (maximize).

So $y = 1200 - 2x$. and $A = x(1200-2x)$.

$= 1200x - 2x^2$

Compute $A' = 1200 - 4x$

critical pt: set $1200 - 4x = 0 \Rightarrow x = 300$.

$A'' = -4 < 0$ loc. max @ $x = 300$.

It's the only critical pt: it must be global max @ $x=300$.

$y = 1200 - 2\times 300 = 600$

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dimensions : 300×600 Choose (b).

Q7. $a(t) = 12t$ on $[0, 10]$.

$$v(t) = \int a(t)\,dt = \int 12t\,dt = 6t^2 + C_1$$

$v(0) = 12 \Rightarrow C_1 = 12$, so $v(t) = 6t^2 + 12$.

$$s(t) = \int v(t)\,dt = \int (6t^2+12)\,dt = 2t^3 + 12t + C_2$$

$s(1) = 15 \Rightarrow 2 + 12 + C_2 = 15 \Rightarrow C_2 = 1$

Then $s(t) = 2t^3 + 12t + 1$

So $s(5) = 2\cdot 5^3 + 12\cdot 5 + 1 = 250 + 60 + 1 = 311$

Choose (d).

Q8. $f(3) = 1$, $f'(3) = -3$. Given $h(x) = \dfrac{2f(x)}{x^2+1}$. $h'(3) = ?$

$$h'(x) = \frac{2f'(x)(x^2+1) - 2f(x)\cdot 2x}{(x^2+1)^2}$$

$$\Rightarrow h'(3) = \frac{2f'(3)\cdot(3^2+1) - 2f(3)\cdot 2\cdot 3}{(3^2+1)^2}$$

$$= \frac{2\cdot(-3)\cdot 10 - 2\cdot 1\cdot 6}{10^2} = \frac{-72}{100}$$

Choose (a)

Q 9. $\quad g(x) = 2\sin(5x)$.

$1^{st} \rightarrow g'(x) = 2 \cdot 5 \cdot \cos(5x)$.

$2^{nd} \rightarrow g''(x) = 2 \cdot 5 \cdot 5 \ (-\sin(5x))$

$3^{rd} \rightarrow g'''(x) = 2 \cdot 5 \cdot 5 \cdot 5 \cdot (-\cos(5x))$

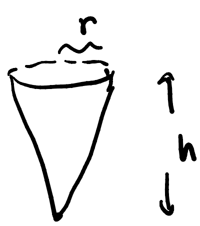
$g^{(4)}(x) = 2 \cdot 5^4 \cdot \sin(5x)$.

$4003 = 4000 + 3$

$g^{(4003)}(x) = 2 \cdot 5^{4003} (-\cos(5x))$.

Choose (e).

Q 10.



$V = \frac{\pi}{3} r^2 h$

use $h = 2r \Rightarrow r = \frac{h}{2}$

$\Rightarrow V = \frac{\pi}{3} \left(\frac{h}{2}\right)^2 h = \frac{\pi}{12} h^3$

differentiate with respect to t : $\frac{dV}{dt} = \frac{\pi}{12} 3h^2 \cdot \frac{dh}{dt}$

$\frac{dV}{dt} = 10$ and $h = 8$

$\Rightarrow \quad 10 = \frac{\pi}{12} \cdot 3 \cdot 8^2 \cdot \frac{dh}{dt}$

$\Rightarrow \quad \frac{dh}{dt} = \frac{120}{3\pi \cdot 8^2} = \frac{10}{16\pi} = \frac{5}{8\pi}$

Choose (b).

Q 11. $f(x) = -3x^2 + 5x + 5$ on $[0, 3]$.

$$\frac{f(b)-f(a)}{b-a} = \frac{f(3)-f(0)}{3-0} = \frac{(-27+15+5)-5}{3} = \frac{-12}{3} = -4$$

$$f'(x) = -6x+5$$

$$\Rightarrow \quad f'(c) = \frac{f(b)-f(a)}{b-a} \quad \Rightarrow \quad -6c+5 = -4$$

$$c = \frac{-9}{-6} = \frac{3}{2}$$

Choose (c).

Q12. $$\ln f(x) = \ln \frac{(x^3+2x)^{400}}{(1+x)^{300}} = \ln (x^3+2x)^{400} - \ln (1+x)^{300}$$

$$= 400 \ln (x^3+2x) - 300 \ln (1+x).$$

differentiate it :

$$\frac{1}{f(x)} \cdot f'(x) = 400 \cdot \frac{1}{x^3+2x} \cdot (3x^2+2) - 300 \cdot \frac{1}{1+x} \cdot 1$$

$$f'(x) = \left[\frac{400(3x^2+2)}{x^3+2x} - \frac{300}{1+x}\right] \cdot \frac{(x^3+2x)^{400}}{(1+x)^{300}}$$

Choose (a).

Q 13. $x = 2t^3 - t^2 + 6$ and $y = -t^3 + \frac{9}{2}t^2 - 6t$.

tangents : $$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-3t^2+9t-6}{6t^2-2t}$$

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horizontal: $\frac{dy}{dx}=0 \Rightarrow -3t^2+9t-6=0.$

$\Rightarrow -3(t^2-3t+2)=0$

$\Rightarrow -3(t-2)(t-1)=0.$

$t=1$ and $t=2$.

vertical: $\frac{dx}{dy}=0 \Rightarrow 6t^2-2t=0$

$\Rightarrow 2(3t^2-t)=0$

$\Rightarrow 2t(3t-1)=0$

$t=0$ and $t=\frac{1}{3}$

Choose (a).

Q14. $\left(\langle \sqrt{10t+5}, e^{4t-8} \rangle\right)'$

$= \langle \frac{1}{2\sqrt{10t+5}} \cdot 10, \ e^{4t-8} \cdot 4 \rangle$

@ $t=2$: $\langle \frac{5}{\sqrt{20+5}}, e^0 \cdot 4 \rangle = \langle 1, 4 \rangle.$

$\frac{\langle 1,4\rangle}{\|\langle 1,4\rangle\|} = \frac{\langle 1,4\rangle}{\sqrt{1+16}} = \langle \frac{1}{\sqrt{17}}, \frac{4}{\sqrt{17}} \rangle.$

Choose (c).

Q15.

① $(ax^3+16x)' = 3ax^2+16$

$(5x^2+6)' = 10x$

they match @ $x=1$.

① $(ax^3+16x)' = 3ax^2+16$. they match @ $x=1$.

$(5x^2+b)' = 10x$

$$\Rightarrow \quad 3a+16=10 \quad \Rightarrow \quad a=-2.$$

② $\left.\begin{array}{l} ax^3+16x \\ 5x^2+b \end{array}\right\}$ match @ $x=1$. ← (continuity).

$$\Rightarrow \quad a+16 = 5+b \quad \Rightarrow \quad b = a+11 = -2+11 = 9$$

Choose (b).

Q 16. $\int_5^9 g(x)\,dx = 4$.

$$\int_5^9 (3-4\cdot g(x))\,dx = \int_5^9 3\,dx - 4\underbrace{\int_5^9 g(x)\,dx}_{4}$$

$$= 3\cdot(9-5) - 4\cdot 4$$

$$= 12 - 16 = -4$$

Choose (d).

Q17. $f(x) = \int_{\tan(x)}^{x} \frac{1}{\sqrt{4+t^3}}\,dt$

$$f'(x) = \frac{1}{\sqrt{4+x^3}} - \frac{1}{\sqrt{4+\tan^3(x)}}\cdot(\tan(x))'$$

$$= \frac{1}{\sqrt{4+x^3}} - \frac{1}{\sqrt{4+\tan^3(x)}}\cdot\sec^2(x).$$

$\sqrt{4+x^3}$ $\quad$ $\sqrt{4+\tan^3(x)}$

Choose (c).

(In general FTC 1 + chain rule : $\frac{d}{dx}\left(\int_{h(x)}^{g(x)} f(t)\,dt\right) = f(g(x))\cdot g'(x) - f(h(x))\cdot h'(x)$)

Q 18.
$$\int_1^2 \left(\frac{9}{x^5} - \frac{2}{x}\right) dx$$

$$= \int_1^2 (9\cdot x^{-5} - 2\, x^{-1})\, dx$$

$$= \left(9\cdot\frac{1}{-4}\cdot x^{-4} - 2\cdot \ln|x|\right)\Big|_1^2$$

$$= \left(\frac{9}{-4}\cdot 2^{-4} - 2\cdot\ln 2\right) - \left(\frac{9}{-4}\cdot 1 - 2\ln(1)\right).$$

$$= -\frac{9}{64} - 2\ln 2 + \frac{9}{4}$$

$$= \frac{-9}{64} + \frac{144}{64} - 2\ln 2 = \frac{135}{64} - 2\ln 2$$

Choose (a).

Q 19.
$$\int \left(3x^2 - 10 + \frac{3}{x^2+1}\right) dx = x^3 - 10x + 3\cdot\arctan(x) + C$$

choose (c)

Q 20.
$$S(4) - S(0) = \int_0^4 v(t)\,dt$$

$$= \int_0^4 (3t-7)\,dt$$

$$= \int_0^4 (3t-7)\,dt$$

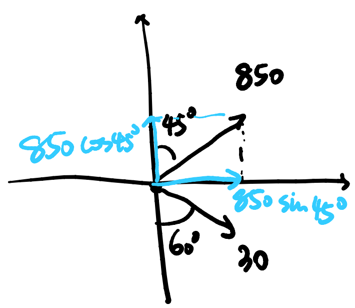
$$= \left(\frac{3t^2}{2} - 7t\right)\Big|_0^4$$

$$= \left(\frac{3\cdot 4^2}{2} - 7\cdot 4\right) - 0$$

$$= 24 - 28 = -4$$

Choose (e).

Q 21.



$$y = 850 \cdot \cos 45^\circ - 30 \cos 60^\circ$$

$$= 850 \cdot \frac{\sqrt{2}}{2} - 30 \cdot \frac{1}{2}$$

$$= 425\sqrt{2} - 15$$

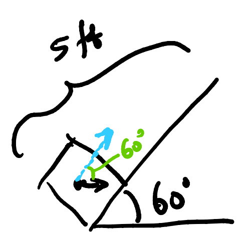
$$x = 850 \sin 45^\circ + 30 \sin 60^\circ$$

$$= 850 \cdot \frac{\sqrt{2}}{2} + 30 \cdot \frac{\sqrt{3}}{2}$$

$$= 425\sqrt{2} + 15\sqrt{3}$$

$$\tan\theta = \frac{y}{x} = \frac{425\sqrt{2} - 15}{425\sqrt{2} + 15\sqrt{3}}$$ choose (d).

Q 22

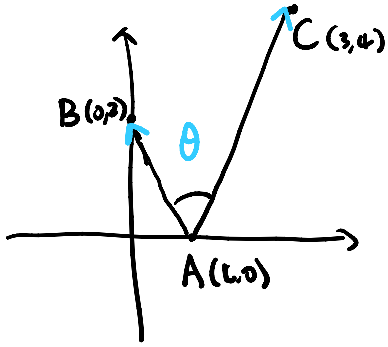


$$F = 20 \cdot \cos 60^\circ = 20 \cdot \frac{1}{2} = 10$$

$$W = 10 \cdot 5 = 50.$$

Choose (a).

Q 23.



$\vec{AB} = \langle -1, 2 \rangle$

$\vec{AC} = \langle 2, 4 \rangle$.

$$\vec{AB} \cdot \vec{AC} = \langle -1, 2 \rangle \cdot \langle 2, 4 \rangle$$
$$= -2 + 8 = 6.$$

Also
$$\vec{AB} \cdot \vec{AC} = \|\vec{AB}\| \cdot \|\vec{AC}\| \cos\theta$$
$$= \sqrt{5} \cdot \sqrt{20} \cos\theta$$

$$\Rightarrow \quad \sqrt{100} \cos\theta = 6 \quad \Rightarrow \quad \cos\theta = \frac{6}{\sqrt{100}} = \frac{6}{10} = \frac{3}{5}$$

$$\theta = \arccos\left(\frac{3}{5}\right)$$

Choose (a).

Q24. differentiate with respect to x:

$$2xy^2 + x^2 \cdot 2y \cdot y' - 3y' = 0.$$

Solve for y': $(x^2 \cdot 2y - 3)y' = -2xy^2$

$$\Rightarrow \quad y' = \frac{-2xy^2}{2x^2y - 3}$$

@ $(1, -3)$:
$$y' = \frac{-2 \cdot 1 \cdot (-3)^2}{2 \cdot 1^2 \cdot (-3) - 3} = \frac{-18}{-9} = 2$$

Choose (d).

Choose (d).

Q25. $\lim_{x\to 3^+} \frac{x^2-2x-7}{x^2-5x+6} = \lim_{x\to 3^+} \frac{x^2-2x-7}{(x-2)(x-3)} = \frac{-4}{0^+} = -\infty$

Choose (a)

Q26. Choose $f(x) = \sqrt[3]{x}$ $a = 27$

$L(x) = f(a) + f'(a)(x-a)$

$= \sqrt[3]{27} + \frac{1}{3}(27)^{-\frac{2}{3}}(x-27).$

$= 3 + \frac{1}{27}(x-27)$

$= \frac{1}{27}x + 2$

$\sqrt[3]{27.2} \approx L(27.2) = \frac{1}{27} \times 27.2 + 2 = \frac{1}{27} \cdot \frac{136}{5} + 2$

$= \frac{136}{135} + 2 = \frac{406}{135}$

choose (c)

Q27.

$$\left.\begin{aligned} &\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x+2 = 4. \\ &f(2) = 2^2 = 4. \end{aligned}\right\} \text{match}$$

$\lim_{x\to 2^+} f(x) = 5$

Choose (a).

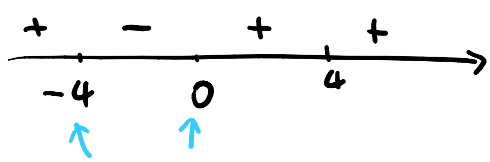
Q 28.

$$\lim_{x \to -\infty} \frac{5e^{2x} - 8e^{-3x}}{3e^{2x} + 2e^{-3x}} = \lim_{x \to -\infty} \frac{0 - 8e^{-3x}}{0 + 2e^{-3x}} = \frac{-8}{2} = -4$$

Choose (c)

Q 29.

$$f''(x) = 3x(x^2 - 16)(x - 4) = 3x(x+4)(x-4)^2$$



sign of f''

$x = -4$ and $x = 0$.

Choose (b)

Q 30.

$$f(x) = \arcsin(e^{4x}).$$

$$f'(x) = \frac{1}{\sqrt{1 - (e^{4x})^2}} \cdot e^{4x} \cdot 4$$

$$= \frac{4e^{4x}}{\sqrt{1 - e^{8x}}}$$

Choose (a)