Week In Review (week 13)

Math 151 - Fall 2022
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Problems:

1. Approximate the area under the curve $y = x^3 - 4$ on $[-2, 6]$ using 4 equal-width rectangles and left endpoints.

2. Find the interval(s) where $f(x)$ is increasing if
$$f'(x) = \frac{(x-4)^3(x+6)^8}{7-x}.$$

3. Evaluate $\lim\limits_{x\to\infty} x \sin\left(\frac{5}{x}\right)$.

4. Use geometry to evaluate $\int_0^2 \left(\sqrt{4-x^2} + 3x\right) dx$.

5. Find the value(s) of c that satisfies the conclusion of the Mean Value Theorem for the function $f(x) = \sqrt{2x}+3$ on the interval $[1/2, 2]$.

6. Find $f(x)$ if $f'(x) = \frac{1}{x^2+1} - 5 - \sin(x)$ and $f(1) = \cos(1)$.

7. Use the limit of right endpoint Riemann sum to express the area of the region under the curve $f(x) = (x^3+5)^2$ on $[1, 6]$.

8. Find the x-coordinate(s) of the inflection points of the function $f(x)$ if $f''(x) = x^3(x^2+16)(x-1)$.

9. Assume that the length from the tip of a cone to the "rim" of it is fixed at 1 inch. The angle of the cone is the only thing that changes. What are the radius and the height of the cone that maximize the volume?

10. Compute the following limits.
    (a) $\lim\limits_{x\to\infty} \left[\ln(2x^2+9) - 3\ln(x+1)\right]$
    (b) $\lim\limits_{x\to 0^+} (1+\sin(x))^{3/x^2}$
    (c) $\lim\limits_{x\to 0^+} (2^x - 1)\csc(x)$

11. Find $f(x)$.
    (a) $f'(x) = 4^x + \sqrt[3]{x^4} - \frac{1}{7x} - \sin(x)$
    (b) $f'(x) = \frac{2x^3 - 7}{x^4}$
    (c) $f'(x) = x^2(8x-3)$ and $f(1) = -1$
    (d) $f'(x) = 4x^3 + \frac{6}{1+x^2} - 7$ and $f(0) = 9$

12. Consider the function $f(x) = \frac{4}{x} + \frac{x}{4} + 2$.
    (a) What is the domain of f?
    (b) Determine the interval(s) where f is increasing and decreasing. Find the x-coordinate(s) of the local extrema.
    (c) Determine the concavity of f.
    (d) If we restrict the function f on the interval $[1, 5]$, find the absolute extrema.

13. A farmer uses 4800 feet of fencing to enclose a rectangular field. He will need one extra side to divide the region into two retangular subregions. What dimensions of the field will maximize the total area?