Problems:

1. A rectangular storage container with an open top is to have a volume of 10 m^3. The length of its base is twice the width. Material for the base costs \$10 per square meter. Material for the sides costs \$6 per square meter. Find the cost of materials for the cheapest such container.

2. Find the base of the rectangle with largest area which can be inscribed in the first quadrant of the ellipse

$$x^2 + \frac{y^2}{4} = 1.$$

   Clearly show that your answer yields maximum area.

3. Find the most general anti-derivative of the following functions.
   (a) $f(x) = \dfrac{5}{\sqrt{1-x^2}} - \dfrac{7+3x-x^4}{x} + \dfrac{1}{1+x^2}$
   (b) $f(x) = 3x^2(x^3+1)$
   (c) $f(x) = \dfrac{2x^2+6}{x^3}$
   (d) $f(x) = \csc(x)(\cot(x) - \csc(x))$
   (e) $f(x) = 7^x + \dfrac{1}{5x^3} + \sqrt[5]{x^3}$

4. Find $f(x)$ of the following.
   (a) $f'(x) = 2(1-x^2)^{-1/2} + e^x$ with $f(0) = 4$
   (b) $f'(x) = 2e^x - 5$ with $f(0) = 1$
   (c) $f''(x) = 5x^4 - 6$ with $f'(0) = 4$ and $f(1) = 2$
   (d) $f''(x) = 20x^3 + 6e^x$ with $f(0) = 4$ and $f(1) = 2$

5. Approximate the area under the graph of $f(x) = 7x - x^2$ from $x = -1$ to $x = 5$ using 6 equal-width subintervals and using right endpoints.

6. Estimate the area under the graph of $f(x) = \sin(x)$ from $x = 0$ to $x = \frac{\pi}{2}$ using 3 approximating rectangles and right endpoints. Sketch the graph and the rectangles. Is the estimate an underestimate or an overestimate?

7. Find an expression for the approximate area under the graph of $f(x) = \sqrt[3]{x^3 - x^2 + 9}$ on the interval $[-2, 7]$ using left endpoints.

8. Find an expression for the actual area under the graph of $f(x) = \dfrac{x^2}{3x-2}$ on the interval $[-5, 2]$ using left endpoints. Do not evaluate the limit.

9. Find an expression for the actual area under the graph of $f(x) = \sqrt{\sin(x)}$ on the interval $[0, \pi]$ using right endpoints. Do not evaluate the limit.

10. Determine a region whose area is equal to the limit

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{4n} \tan\frac{i\pi}{4n}.$$

   Do not evaluate the limit.