Week In Review (week 11)

Math 151 - Fall 2022
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Problems:

1. Given $f(x) = x^4 - 2x^2 + 3$.
   (a) Find the intervals on which f is increasing or decreasing.
   (b) Find the local max and local min values of f.
   (c) Find the intervals of concavity and the inflection points.
2. Given $f(x) = x^2 \ln(x)$.
   (a) Find the intervals on which f is increasing or decreasing.
   (b) Find the local max and local min values of f.
   (c) Find the intervals of concavity and the inflection points.
3. Sketch the graph of $f(x)$ that satisfies all of the following conditions.
   (a) $f'(0) = f'(2) = f'(4) = 0$.
   (b) $f'(x) > 0$ if $x < 0$ or $2 < x < 4$, and $f'(x) < 0$ if $0 < x < 2$ or $x > 4$.
   (c) $f''(x) > 0$ if $1 < x < 3$, and $f''(x) < 0$ if $x < 1$ or $x > 3$.
4. Sketch the graph of $f(x)$ that satisfies all of the following conditions.
   (a) $f'(5) = 0$
   (b) $f'(x) < 0$ when $x < 5$ and $f'(x) > 0$ when $x > 5$.
   (c) $f''(2) = f''(8) = 0$
   (d) $f''(x) < 0$ when $x < 2$ or $x > 8$, and $f''(x) > 0$ when $2 < x < 8$.
   (e) $\lim_{x\to\infty} f(x) = 3$ and $\lim_{x\to-\infty} f(x) = 3$.
5. Given $f(x) = 1 + \frac{1}{x} - \frac{1}{x^2}$.
   (a) Find the vertical and horizontal asymptotes.
   (b) Find the intervals on which f is increasing or decreasing.
   (c) Find the local max and local min values of f.
   (d) Find the intervals of concavity and the inflection points.
   (e) Use the previous information to sketch the graph of f.
6. Compute the following limits.
   (a) $\lim_{x\to 0} \frac{\arccos(x) - \frac{\pi}{2}}{3x}$
   (b) $\lim_{x\to 1} \frac{e^{3x-3} + x^3 - 2}{5\ln(x) + 4x - 4}$
   (c) $\lim_{x\to\infty} x^3 e^{-x^3}$
   (d) $\lim_{x\to\infty} x \sin\left(\frac{\pi}{x}\right)$
   (e) $\lim_{x\to 0^+} x^3 \ln(x)$
   (f) $\lim_{x\to 0} \left(\frac{1}{x^2} - \frac{1}{e^x - 1}\right)$
   (g) $\lim_{x\to\infty} \left(\frac{2x^2}{2x+1} - \frac{x^2}{x+3}\right)$
   (h) $\lim_{x\to 0^+} (3x + 1)^{\csc(x)}$
   (i) $\lim_{x\to\infty} (1 + x + x^2)^{\frac{1}{\ln(x)}}$
   (j) $\lim_{x\to\infty} \left(1 + \frac{3}{x}\right)^{5x}$