6 12 An insurance company has an obligation to pay the medical costs for a claimant. Average annual claims costs today are 6750, and medical inflation is expected to be 3.25% per year. The claimant is expected to live an additional 16 years. Claim payments are made at yearly intervals, with the first claim payment to be made one year from today. Find the present value of the obligation if the annual interest rate is 5.8%.

Method 1: Basic Principals

$$PV = 6750(1.0325)v + 6750(1.0325v)^2 + \cdots + 6750(1.0325v)^{16}$$

$$= 6750(1.0325)v\left[\frac{1-(1.0325v)^{16}}{1-1.0325v}\right] \qquad \text{where } v = \frac{1}{1+0.058}$$

$$= \boxed{88329.18}$$

Method 2: Formula for Geometrically Varying Annuity

$$PV = 6750(1.0325)\left[\frac{1-\left(\frac{1.0325}{1.058}\right)^{16}}{0.058-0.0325}\right] = \boxed{88329.18}$$

Method 3: Level Annuity Trick

$$PV = 6750\left[1.0325v + (1.0325v)^2 + \cdots + (1.0325v)^{16}\right]$$

Find j such that $1.0325v = \frac{1.0325}{1.058} = \frac{1}{1+j} = v_j$

$$1.0325 + 1.0325j = 1.058$$

$$j = 0.0246973337$$

So now we have

$$PV = 6750\left[v_j + v_j^2 + \cdots + v_j^{16}\right] = 6750\,a_{\overline{16}|j}$$

N = 16 I/Y = 2.4697337 PV = $\boxed{\text{CPT}}$ PMT = −6750 FV = 0

$$PV = \boxed{88329.18}$$

Note: If $\frac{1+k}{1+i} > 1$, then find j such that $\frac{1+k}{1+i} = 1+j$ and find FV.

7 Ø. An annuity pays 1 at the end of the first year. The payments increase by 2% for the next 7 years, the payments increase by 9% for the following 10 years, and then the payments increase by 11% forever. Calculate the present value of this annuity given an annual effective interest rate of 15% .

Time	0	1	2	3	. . .	8	9	10	. . .	18	. . .
Payment		1	$1(1.07)$	$1(1.07)^2$		$1(1.07)^7$	$(1.07)^7(1.09)$	$(1.07)^7(1.09)^2$	. . .	$(1.07)^7(1.09)^{10}$	. . .

Time	. . .	19	20	. . .
Payment		$(1.07)^7(1.09)^{10}(1.11)$	$(1.07)^7(1.09)^{10}(1.11)^2$	. . .

Method 1: Formula

$$PV = \underbrace{\frac{1-\left(\frac{1.07}{1.15}\right)^7}{0.15-0.07}}_{\text{PMTS 1-7}} + \underbrace{(1.07)^7\left[\frac{1-\left(\frac{1.09}{1.15}\right)^{10}}{0.15-0.09}\right]v^7}_{\text{PMTS 8-17}} + \underbrace{(1.07)^7(1.09)^{10}\left[\frac{1-\cancel{\left(\frac{1.11}{1.15}\right)^{\infty}}^{\,0}}{0.15-0.11}\right]v^{17}}_{\text{PMTS } 18\to\infty}$$

$$PV = 4.95409 + 4.17363 + 8.83137 = \boxed{17.95909}$$

Method 2: Level Annuities Trick

$$PV_1 = v + 1.07v^2 + 1.07^2v^3 + \cdots + 1.07^7v^8.$$

$$(\text{Pmts 1-8}) = \frac{1}{1.07}\left[1.07v + (1.07v)^2 + \cdots + (1.07v)^7\right]$$

Need j such that $\frac{1.07}{1.15} = \frac{1}{1+j} \Rightarrow j = 0.0747666$

$N = 8$
$I/Y = 7.47666$
$PV = \boxed{CPT} \rightarrow PV_1 = 5.47902$
$PMT = 1/1.07$
$FV = 0$

$$PV_2 @ t=8 = (1.07)^7\left[1.09v + (1.09v)^2 + \cdots + (1.09v)^{10}\right]$$

(PMTS 9–18)

$$\frac{1.09}{1.15} = \frac{1}{1+j} \Rightarrow j = 0.05504587$$

$N = 10$
$I/Y = 5.504587$
$PV = \boxed{CPT} \rightarrow PV_2 @ t=8 = 12.10112$
$PMT = (1.07)^7$
$FV = 0$

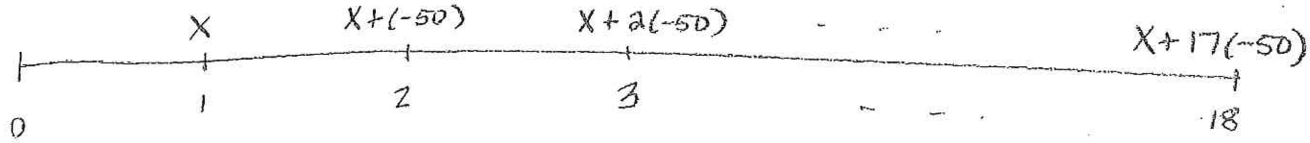
$$PV_3 @ t=18 = (1.07)^7(1.09)^{10}(1.11)\left[\frac{1}{0.15-0.11}\right]$$

PMTS $19 \to \infty$

$$= 105.49076$$

$$PV = PV_1 + PV_2 v^8 + PV_3 v^{18} = \boxed{17.95909}$$

8. An annuity-immediate paying X at the end of first year, with each subsequent payment decreased by 50 for the following 17 years (for a total of 18 payments), has a present value of 35,985. If $i = 0.7\%$, calculate X.



$$PV = Pa_{\overline{n}|} + Q\left[\frac{a_{\overline{n}|} - nv^n}{i}\right]$$

$$35985 = Xa_{\overline{18}|} + (-50)\left[\frac{a_{\overline{18}|} - 18v^{18}}{0.007}\right]$$

$$\boxed{X = 2550.35}$$

$\underline{a_{\overline{18}|}:}$

N=18   I/Y=0.7   PV=[CPT]   PMT=−1   FV=0

$a_{\overline{18}|} = 16.856860872$

[STO] [1] (Store this value in register 1 to save time)

Keystrokes (assuming BA II is set to AOS)

1.007 [y^x] 18 [+|−] [=] [×] 18 [+|−] [=] [+] [RCL] [1] [=] [÷] 0.007 [=]

[×] 50 [=] [+] 35985 [=] [÷] [RCL] [1] [=]

9 9. 3000 is deposited into Fund X, which earns an annual effective rate of 7%. At the end of each year, the interest earned plus an additional 300 is withdrawn from the fund. At the end of the tenth year, the fund is depleted. The annual withdrawals of interest and principal are deposited into Fund Y, which earns an annual effective rate of 9%. Determine the accumulated value of Fund Y at the end of year 10.

Fund Y

	0	1	2	3	. . .	10
		3000(0.07) + 300	2700(0.07) + 300	2400(0.07) + 300	. . .	300(0.07) + 300
Same as:		210 + 300	189 + 300	168 + 300	. . .	21 + 300
Same as		210 + 300	210 − 21 + 300	210 − 21(2) + 300	. . .	210 − 21(9) + 300

Find PV

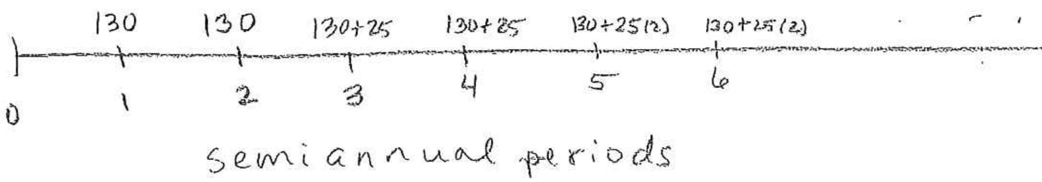
$$PV = 300a_{\overline{10}|} + 210a_{\overline{10}|} + (-21)\left[\frac{a_{\overline{10}|} - 10v^{10}}{0.09}\right]$$

$$= 2761.177180$$

Now accumullate to t = 10

$$2761.17718(1.09)^{10} = \boxed{6536.71}$$

10 9. A perpetuity immediate pays out 130 semiannually for the first year. From then on the payments increase by 25 every year. Given the annual effective interest rate is 4%, find the purchase price of this perpetuity.



Key: Treat as two arithmetically increasing perpetuities.

$$PV = \left[\frac{130}{0.04} + \frac{25}{(0.04)^2}\right](1.04)^{0.5} + \frac{130}{0.04} + \frac{25}{(0.04)^2}$$

$$\boxed{PV = 38,123.80}$$