This is not a comprehensive review for Exam 2. These are just 5 problems that cover only a portion of the material taught in Chapter 4.

1. Find the present value of a 15-year decreasing annuity-immediate paying 150,000 the first year and decreasing by 10,000 each year thereafter. The effective annual interest rate of 4.5%. **Answer: 946,767.616**

2. An investor is considering the purchase of 500 ordinary shares in a company. This company pays dividends at the end of each year. The next payment is one year from now and it is $3 per share. The investor believes that each subsequent payment per share will increase by $1 each year forever. Calculate the present value of this dividend stream at a nominal rate of interest of 6.8% per annum compounded semiannually. **Answer:$126,236.78**

3. The force of interest at time t is $\delta_t = \frac{t^3}{10}$. Find the accumulated value of a four-year continuous annuity which has a rate of payments at time t of $5t^3$. Give an exact answer. **Answer:** $50e^{6.4} - 50$

4. An annuity provides for 10 annuals payments, the first payment one year from now being $2600. The payments increase in such a way that each payment is 3% greater than the previous one. The annual effective rate of interest is 4%. Find the present value of this annuity. **Answer: $23,945.54**

5. Find the accumulated value of an annuity-due in which annual payments of $43,000 paid monthly for 10 years are made to an account paying an annual effective rate of discount of 5.6%. **Answer: $582,967.19**

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Chapter 4 Selected Practice Problems

1) Arithmetic Progression

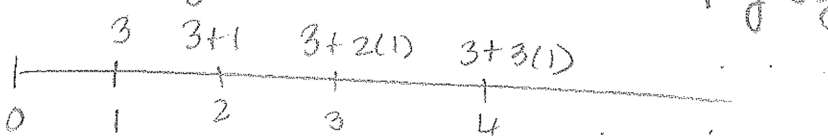
$$PV = Pa_{\overline{n|}} + Q\left[\frac{a_{\overline{n|}} - nv^n}{i}\right]$$

$$= 150000\, a_{\overline{15|}.045} + (-10000)\left[\frac{a_{\overline{15|}.045} - 15v^{15}}{0.045}\right]$$

$$= 1{,}610{,}931.859 - 664{,}164.2426$$

$$= \boxed{946{,}767.6164}$$

2) Find PV of 1 share and then multiply by 500.



Perpetuity varying in arithmetic progression

$$PV = \frac{P}{i} + \frac{Q}{i^2} = \frac{3}{i} + \frac{1}{i^2} \quad \text{where } i = \left(1+\frac{0.068}{2}\right)^2 - 1$$

PV = 252.473557 for 1 share

so $\boxed{\$126{,}236.78 \text{ for 500 shares}}$

3) $FV = \int_0^4 f(t) e^{\int_t^4 \delta_r \, dr} \, dt$

$= \int_0^4 5t^3 e^{\int_t^4 \frac{1}{10} r^3 \, dr} \, dt$

$= \int_0^4 5t^3 e^{\frac{1}{40} r^4 \Big|_t^4} \, dt$

$= \int_0^4 5t^3 e^{\frac{4^4}{40} - \frac{t^4}{40}} \, dt$

$u = -\frac{t^4}{40} + \frac{64}{10}$

$du = \frac{-t^3}{10} \, dt$

$= -50 \int_{t=0}^{t=4} e^u \, du$

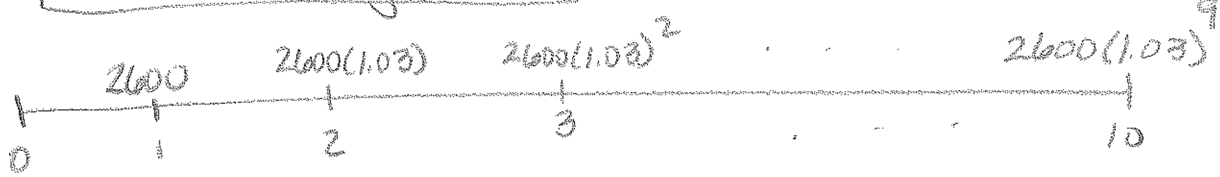
$= -50 e^{-\frac{t^4}{40} + \frac{64}{10}} \Big|_0^4$

$= -50 \left(e^{-\frac{4^4}{40} + \frac{64}{10}} - e^{6.4} \right)$

$= -50(1 - e^{6.4})$

$= \boxed{50e^{6.4} - 50}$

4) $\boxed{\text{Geometric Progression}}$



$i = 0.04$

$$PV = 2600v + 2600(1.03)v^2 + 2600(1.03)^2 v^3 + \cdots + 2600(1.03)^9 v^{10}$$

$$= 2600v\left(1 + 1.03v + (1.03v)^2 + \cdots + (1.03v)^9\right)$$

$$= 2600v\left(\frac{1-(1.03v)^{10}}{1-1.03v}\right) \quad \text{where } v = \frac{1}{1.04}$$

$$= \boxed{\$23,945.54}$$

Alternative

If 1st pmt is 1 and pmts then have geom. progression w/ common ratio $1+k$, then

$$PV = \frac{1-\left(\frac{1+k}{1+i}\right)^n}{i-k}$$

Applied here, we have

$$2600\left[\frac{1-\left(\frac{1+0.03}{1+0.04}\right)^{10}}{0.04-0.03}\right] = \boxed{23,945.54}$$

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5) $FV = 43000\,\ddot{s}^{(12)}_{\overline{10|}} = 43000\left[\dfrac{(1+i)^{10}-1}{d^{(12)}}\right]$

Find i

$(1-d)^{-1} - 1 = i$

$\dfrac{1}{1-0.056} - 1 = i$

$i = 0.05932203$

$= 43000\left[\dfrac{(1.05932203)^{10}-1}{0.05749095}\right]$

$= \boxed{\$582{,}967.19}$

Find $d^{(12)}$

$1-0.056 = \left(1-\dfrac{d^{(12)}}{12}\right)^{12}$

$d^{(12)} = 0.05749095$