Interest

- Simple Interest: $A(t) = A(0)(1 + it)$ where t is in years, $A(0)$ represents the initial principal, and i is the simple interest rate

- Compound Interest

  - Effective Rate of Interest, i: $A(t) = A(0)(1 + i)^t$ where $A(t)$ is the future value at time t and $A(0)$ is the present value or initial principal

  - To find a present value at time 0 of some given future value at time t assuming an effective rate of i, divide by $(1 + i)^t$. Alternatively, if we let $v = \frac{1}{1+i}$, then to discount a future value back t years, multiply by v^t.

  - Nominal Rate of Interest, $i^{(m)}$, compounded m^{th}-ly: $A(t) = A(0)\left(1 + \dfrac{i^{(m)}}{m}\right)^{mt}$

  - Force of Interest, δ: $\delta = \ln(1 + i)$ and $A(t) = A(0)e^{\delta t}$

- Force of Interest (in general): $\delta_t = \dfrac{a'(t)}{a(t)}$ where $a(t)$ represents the accumulation function, and $A(t) = A(0)e^{\int_0^t \delta_r dr}$

- Simple Discount: $A(t) = A(0)(1 - dt)^{-1}$ where t is in years, $A(0)$ represents the initial principal, and d is the simple discount rate

- Compound Discount: To accumulate forward one period, multiply by $(1 - d)^{-1}$ where d is the effective rate per period or multiply by $\left(1 - \dfrac{d^{(m)}}{m}\right)^{-m}$ where $d^{(m)}$ is the nominal rate of discount compounded m^{th}-ly.

- Real Rate of Return, i': $1 + i' = \dfrac{1 + i}{1 + r}$ where i is the annual effective rate of interest and r is the rate of inflation. Also, $i' = \dfrac{i - r}{1 + r}$.

1. Find the simple interest on a $1500 investment made for 5 years at an interest rate of 3.5%/year. Also, what is the accumulated amount?

2. Molly would like to have $3,500 at the end of 4 years. How much should she deposit today into an account earning simple discount at a rate of 6% to achieve this goal?

3. Eight months ago, Kira borrowed some money from Benjamin, and now she owes him a total of $1,001.72. If Benjamin charged her 8% simple interest, how much did Kira originally borrow from him?

4. David deposits $250 today into an account that earns simple discount at a rate of 9%. Find the accumulated value at the end of 5 months.

5. What nominal rate of interest compounded monthly is equivalent to a nominal rate of discount of 6.8% compounded semiannually?

6. What annual effective rate of discount is equivalent to a nominal rate of interest of 12% compounded quarterly?

7. Bob needs 10,000 fifteen years from now to pay for a graduation present. Find the amount he needs to deposit now to achieve this goal, assuming

   (a) an annual effective interest rate of 4%.

   (b) an annual effective discount rate of 4%.

8. Find the accumulated amount after 6 years if $10,000 is invested at 5% interest per year compounded semiannually.

9. Ed wants to buy a boat in 10 years. How much money should he deposit now into a savings account paying 8%/year compounded monthly if he wants to have $15,000 in the account at the end of 10 years?

10. You put $3,000 into an account, and 4 years later the account had $3,753.15. If the account earned interest compounded daily, what was the nominal annual interest rate?

11. Referring to the previous problem what annual effective rate of discount was earned?

12. You make an investment where you pay 6,000 now and get 7,000 back in 5 years. What nominal interest rate convertible quarterly did you earn?

13. Find the present value of $49,158.60 due in 5 years at a discount rate of 10% per year compounded quarterly.

14. Bank A offers a 2500 CD over 5 years with an annual effective interest rate of 7% with a bonus of 2% of the face value at expiration. Bank B offers CDs of the same amount over 5 years, but without the bonus. What yield rate would Bank B have to offer for these CDs to be equivalent?

15. Fund P accrues simple interest at 4% per annum. Fund Q accrues simple interest at $j\%$ per annum. Smith invests 100 in fund P and 118.50 in fund Q. The accumulated amounts are equal in 5 years. Find j.

16. Fred deposits 10 into a fund today and 20 fifteen years later. Interest is credited at a nominal rate of discount of d compounded quarterly for the first 10 years, and at a nominal rate of interest of 6% compounded semiannually thereafter. The accumulated balance in the fund at the end of 30 years is 100. Calculate d.

17. What annual effective rate of interest is equivalent to a constant force of interest of 5%?

18. A fund quotes annual force of interest as follows:

year	1	2	3	4	5
δ_t	6%	4%	5%	7%	8%

A deposit of K accumulates to 1000 over 5 years. What is K?

19. Find the present value of a payment of $3,000 due in 10 years if the force of interest is 7%.

20. An account is created that earns a force of interest of $\delta_t = 0.3 - 0.05t$ for $0 \le t \le 6$ and a constant force of interest of 5% thereafter. If $500 is invested in this account at time $t = 4$, find the accumulated value at time $t = 10$.

21. Suppose an account earning interest in the same manner as the previous exercise is worth $700 at time $t = 8$. What would the present value of this amount be at time $t = 2$?

22. A book club offers a one-year membership at a cost of $50 with renewal the following year at $58. This club also offers a two-year membership for $95. What is the effective annual interest rate that makes the two-year membership equivalent to two successive one-year memberships?

23. If the annual effective rate of interest is 4% and the inflation rate is 1.5%, find the real rate of return.

24. Carl will receive $50 in one year and $150 in two years. If the real rate of return is 7% and the inflation rate is 3%, find the present value of these payments.

Answers

1. Interest: $262.50; Accumulated value: $1762.50

2. $2660

3. $951

4. $259.74

5. 6.9383%

6. 11.1513%

7. a) $5552.65; b) $5420.86

8. $13,448.89

9. $6757.85

10. 5.6000%

11. 5.4457%

12. 3.0949%

13. $29,627.28

14. 7.3034%

15. $j\% = 0.2532\%$

16. 4.5318%

17. 5.1271%

18. $740.82

19. $1489.76

20. $674.93

21. $424.57

22. 28.8889%

23. 2.4631%

24. $168.86