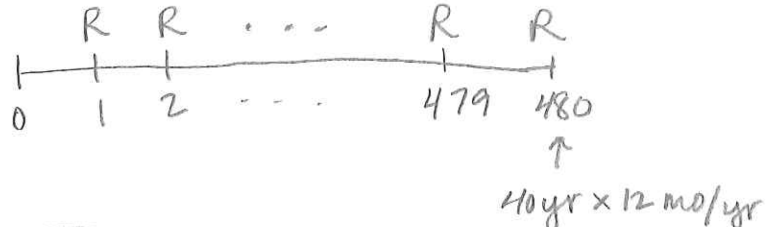
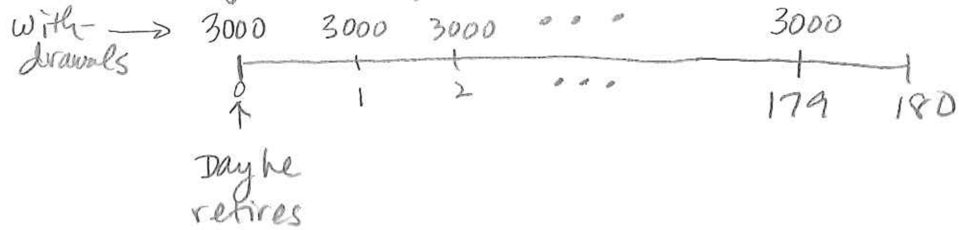
11)



Part (a)



Step 1: Find amt needed in the acct on the day he retires to be able to receive 180 pmts of 3000 each at the beginning of each month.

$$PV = 3000\,\ddot{a}_{\overline{180}|\frac{0.0625}{12}}$$

$$= 3000\left(\frac{\dfrac{1 - v^{180}}{0.0625/12}}{1+0.0625/12}\right)$$

$$= \$351{,}708.0215$$

IVM
Set to BGN
N = 180
I/Y = 6.25/12
PV = CPT
PMT = 3000
FV = 0

Step 2: The value found in Step 1 becomes the FV of the annuity-immediate with 480 monthly pmts of R.

$$R\,s_{\overline{480}|\frac{0.0625}{12}} = 351{,}708.0215$$

$$R\left[\frac{\left(1+\frac{.0625}{12}\right)^{480} - 1}{\frac{.0625}{12}}\right] = 351{,}708.0215$$

$$\boxed{R = \$164.97}$$

IVM
Set back to END
N = 480
I/Y = 6.25/12
PV = 0
PMT = CPT
FV = 351708.0215

11b) Benjamin's total deposit = ($164.97)(480)

= **$79,185.60**

11c) Interest earned over life of the acct = (What Ben received) − (What Ben put in)

= (3000 · 180) − (164.97 · 480)

= **$460,814.40**

Alternatively, look at amt of interest earned while making deposits and while receiving withdrawals separately, and then add these amts together:

<u>Interest while saving</u>

FV − total deposit

= 351708.02 − (164.97 · 480)

= $272,522.42

<u>Interest while receiving withdrawals</u>

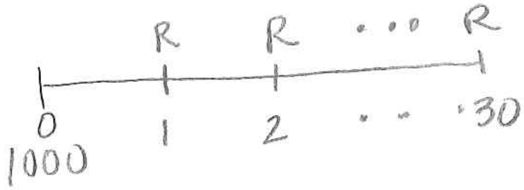
Amt Received − Amt acct started with

= (3000 · 180) − 351708.02

= $188,291.98

Add these: $272 522.42 + $188,291.98 = **$460,814.40**

12) Step 1: Find PMT under original plan.



$$R a_{\overline{30}|.08} = 1000$$

$$R\left(\frac{1-v^{30}}{0.08}\right) = 1000$$

$$R = \$88.83$$

TVM
N=30
I/Y=8
PV=1000
PMT=CPT
FV=0

Step 2: Find PMT if paid back in 20 years.

$$R a_{\overline{20}|.08} = 1000$$

$$R\left(\frac{1-v^{20}}{0.08}\right) = 1000$$

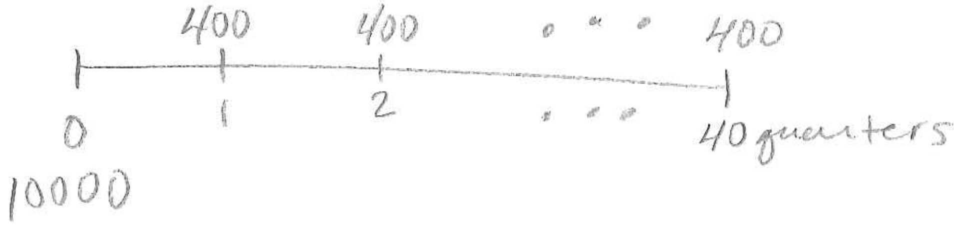
$$R = \$101.85$$

TVM
N=20
I/Y=8
PV=1000
PMT=CPT
FV=0

Step 3: Find the difference in PMT amounts.

101.85 − 88.83 = $13.02 more should be paid each year to clear the debt in 20 yr.

13) Step 1: Find $i^{(4)}$.



Must use a financial calculator (or a calculator able to solve for j in $10000 = 400\left(\frac{1-\left(\frac{1}{1+j}\right)^{40}}{j}\right)$)

N = 40 I/Y = CPT PV = −10000 PMT = 400 FV = 0

$$2.524384862 = \frac{i^{(4)}}{4} \quad \text{(as a percent)}$$

$(\Rightarrow i^{(4)} = 10.09753945)$

Step 2: Convert $i^{(4)}$ to $i^{(12)}$.

$$\left(1+\frac{i^{(12)}}{12}\right)^{12} = \left(1+\frac{i^{(4)}}{4}\right)^{4}$$

$$1+\frac{i^{(12)}}{12} = (1+0.02524384862)^{\frac{4}{12}}$$

$$i^{(12)} = 12\left[(1.0252438\ldots)^{\frac{1}{3}} - 1\right]$$

$$\boxed{i^{(12)} = 10.0137\%}$$

Alternatively, use ICONV to change $i^{(4)}$ to i and then i to $i^{(12)}$.
(i ↑ Effective annual rate.)

14) $21.80\, s_{\overline{10}|i} = 19.28\, a_{\overline{\infty}|i} = X$

$$21.80\left(\frac{(1+i)^{10}-1}{i}\right) = 19.28\left(\frac{1}{i}\right)$$ ← multiply both sides by i

$$21.80(1+i)^{10} - 21.80 = 19.28$$ ← solve for i

$$(1+i)^{10} = \frac{41.08}{21.80}$$

$$i = \left(\frac{41.08}{21.80}\right)^{\frac{1}{10}} - 1$$

Now find X: $X = 19.28\, a_{\overline{\infty}|i}$

$$= 19.28\left(\frac{1}{\left(\frac{41.08}{21.80}\right)^{\frac{1}{10}} - 1}\right)$$

$$= \boxed{\$294.75}$$

Check

Using $i = \left(\frac{41.08}{21.80}\right)^{\frac{1}{10}} - 1$,

$$21.80\, s_{\overline{10}|i} = 21.80\left(\frac{(1+i)^{10} - 1}{i}\right)$$

$$= \$294.75$$ ✓

15) Option 1

Option 2

Set PV of option 1 = PV of option 2

$$15000 - 8000v^5 = 1400 + X a_{\overline{5}|0.10}$$

$$8632.63 = X a_{\overline{5}|0.1} \leftarrow$$

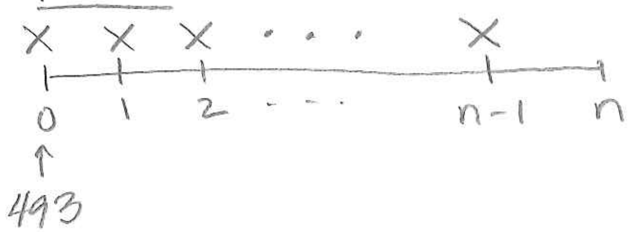
TVM
N = 5
I/Y = 10
PV = 8632.60294
PMT = CPT
FV = 0

$$X = \frac{8632.63}{\left[\frac{1-v^5}{0.1}\right]}$$

$$\boxed{X = \$2277.27}$$

16)

Rick

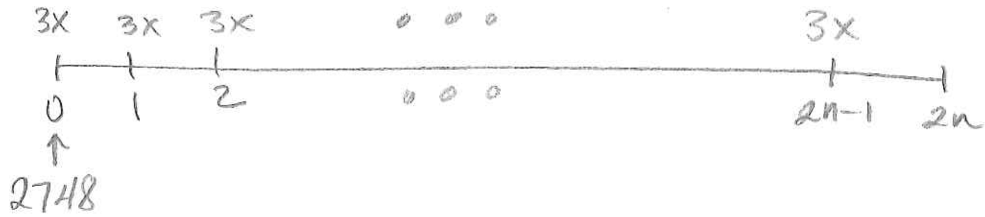


$$\Rightarrow X\ddot{a}_{\overline{n}|i} = 493$$

$$X\left(\frac{1-v^n}{d}\right) = 493$$

$$X = 493 \cdot \frac{d}{1-v^n}$$

Alice



$$3X\ddot{a}_{\overline{2n}|} = 2748$$

so $X = 493 \cdot \dfrac{d}{1-v^n}$ and $3X\ddot{a}_{\overline{2n}|} = 2748$

Now substitute:

$$3\left(493 \cdot \frac{d}{1-v^n}\right)\left(\frac{1-v^{2n}}{d}\right) = 2748$$

$$\frac{1-v^{2n}}{1-v^n} = \frac{916}{493}$$

$$\frac{(1+v^n)(1-v^n)}{1-v^n} = \frac{916}{493}$$

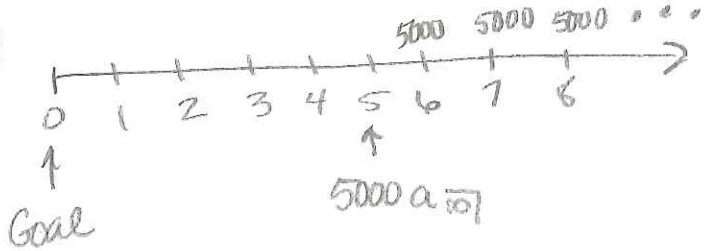
$$1+v^n = \frac{916}{493}$$

$$\boxed{v^n = \frac{423}{493}}$$

$v^n = 1 \Rightarrow$
$v = 1 \Rightarrow$
$\frac{1}{1+i} = 1 \Rightarrow i = 0$
so assume $v^n \neq 1$.
(This allows us to cancel $1-v^n$)

(See last page for alternative solution.)

17)

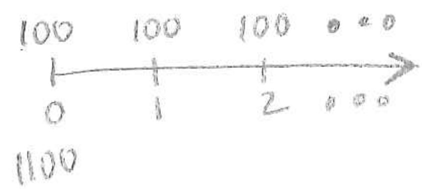


$$PV = 5000\, a_{\overline{\infty}|}\, v^5 \quad \text{where } v = \frac{1}{1+\frac{0.06}{2}}$$

$$= 5000\left(\frac{1}{.06/2}\right) v^5$$

$$= \boxed{\$143{,}768.13}$$

18) Step 1: Use 1st two sentences to find i:



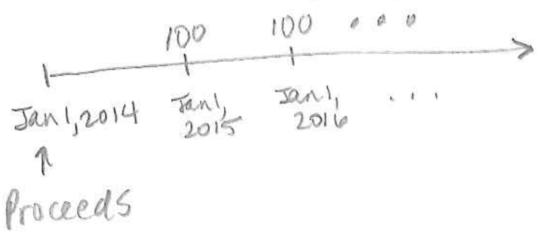
$$1100 = 100\,\ddot{a}_{\overline{\infty}|} = 100 \cdot \frac{1}{d}$$

$$11 = \frac{1}{d}$$

$$d = \frac{1}{11} \Rightarrow i = \frac{d}{1-d} = \frac{\frac{1}{11}}{1-\frac{1}{11}} = \frac{1}{10}$$

So $i = 0.10$.

Step 2: Find the amt of proceeds from sale of remaining pmts on Jan 1, 2014.

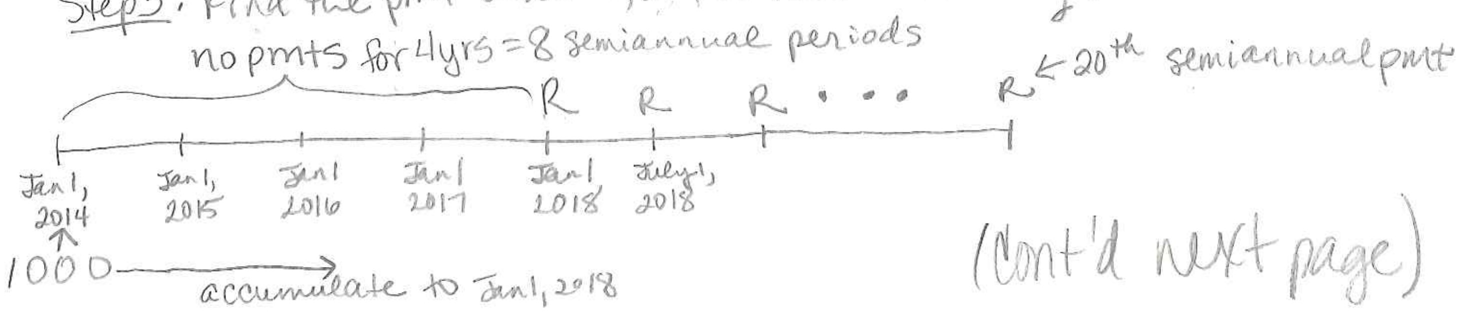


$$\text{Proceeds on Jan 1, 2014} = 100\, a_{\overline{\infty}|} = 100\left(\frac{1}{i}\right)$$

$$= 100\left(\frac{1}{0.10}\right)$$

$$= 1000$$

Step 3: Find the pmt amt for the new annuity.



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#18 cont'd

Equation of Value using Jan 1, 2018 as comparison date:

← I edited the problem to say the interest rate in use starting Jan 1, 2014 is 0.75i compounded semiannually.

$$1000\left(1+\frac{0.075}{2}\right)^8 = R\ddot{a}_{\overline{20}|\frac{0.075}{2}}$$

$$1342.4708 = R\left(\frac{1-v^{20}}{d}\right) \text{ where } v=\frac{1}{1+\frac{0.075}{2}}$$

$$\text{and } d=\frac{0.075/2}{1+0.075/2}$$

R = 93.12

TVM

Set to BGN

N = 20

I/Y = 7.5/2

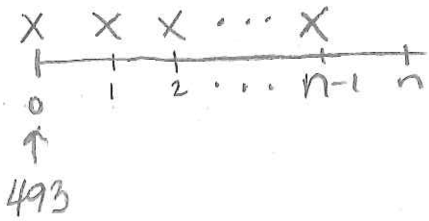
PV = −1342.470784

PMT = CPT → **93.12**

FV = 0

Alternative Soln for #16

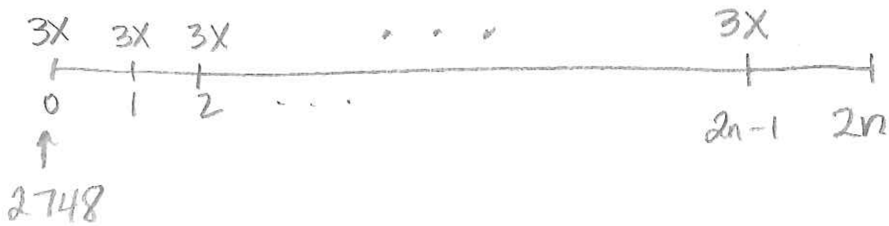
16) Rick



$$X\ddot{a}_{\overline{n}|i} = 493$$

$$\Rightarrow \quad X = \frac{493}{\ddot{a}_{\overline{n}|i}}$$

Alice



$$3X\ddot{a}_{\overline{2n}|i} = 2748$$

Substitute

$$3\left(\frac{493}{\ddot{a}_{\overline{n}|i}}\right)\ddot{a}_{\overline{2n}|i} = 2748$$

$$\ddot{a}_{\overline{2n}|i} = \frac{916}{493}\ddot{a}_{\overline{n}|i}$$

$$\frac{1-v^{2n}}{d} = \frac{916}{493}\left(\frac{1-v^n}{d}\right) \quad \leftarrow \text{multiply both sides by } d$$

$$1-v^{2n} = \frac{916}{493} - \frac{916}{493}v^n \quad \leftarrow \text{quadratic in } v^n$$

$$0 = v^{2n} - \frac{916}{493}v^n + \frac{423}{493}$$

$$\Rightarrow v^n = \frac{\frac{916}{493} \pm \sqrt{\left(-\frac{916}{493}\right)^2 - 4(1)\left(\frac{423}{493}\right)}}{2(1)} = \frac{\frac{916}{493} \pm \frac{70}{493}}{2}$$

$$\Rightarrow v^n = 1 \text{ or } \boxed{v^n = \frac{423}{493}} \quad (\text{Since } v^n = 1 \Rightarrow i = 0)$$

ONLY