

Annuities

- Present value of an annuity-immediate: $a_{\overline{n}|} = \frac{1 - v^n}{i}$
- Present value of an annuity-due: $\ddot{a}_{\overline{n}|} = \frac{1 - v^n}{d}$
- Accumulated value of an annuity-immediate: $s_{\overline{n}|} = \frac{(1+i)^n - 1}{i}$
- Accumulated value of an annuity-due: $\ddot{s}_{\overline{n}|} = \frac{(1+i)^n - 1}{d}$
- Present value of a perpetuity-immediate: $a_{\infty} = \frac{1}{i}$
- Present value of a perpetuity-due: $\ddot{a}_{\infty} = \frac{1}{d}$

NOTE: All of the above formulas assume that i and d are effective rates per period.

1. Which of the following expressions does not represent a definition for $a_{\overline{n}|}$?

A) $v^n \left[\frac{(1+i)^n - 1}{i} \right]$ B) $\frac{1 - v^n}{i}$ C) $v + v^2 + \dots + v^n$ D) $v \left(\frac{1 - v^n}{1 - v} \right)$ E) $\frac{s_{\overline{n}|}}{(1+i)^{n-1}}$

2. Find the present value of regular payments of 200 to be made at the end of each quarter for the next 15 years if the rate of interest is 4% per year compounded quarterly.
3. Find the accumulated value at the end of 20 years of an annuity with 40 payments of 150 made at the beginning of each six-month period if the interest rate is 6% per year compounded semiannually.
4. Consider an annuity with 10 consecutive payments of 1 per period. Using actuarial notation, give six different representations of the value of this annuity at the time that the 7th payment is made.
5. An account receives payments of \$750 at times 0, 1 year, 2 years, etc., with the final payment at time 20 years. Find the value of this annuity at time 0 if the annual effective rate of interest is 7%. *Show that this problem can be worked two ways, one by treating this as an annuity-due, and another by using a formula for an annuity-immediate.*
6. Find the present value of a 6-year deferred 10-year annuity-immediate with semiannual payments of 100 if the nominal interest rate convertible semiannually is 10%.
7. At what nominal rate of interest, convertible monthly, is 30,000 the present value of 1,350 paid at the beginning of each month for two years?
8. Find the accumulated value of an annuity-immediate with 30 payments of 600 per period if the effective rate of interest per period is 5% for the first 18 periods and 3% for the following 12 periods.
9. Jada wants to take a trip to Europe. She plans to make payments at the end of each quarter to an account paying interest at an annual effective rate of 8.25%. How much should each payment be so that she has \$4,500 at the end of 5 years? How much interest will Jada earn?
10. Kira opened an account paying interest at 5.25%/year compounded monthly with \$100 and plans to add \$50 at the end of each month until she has at least \$45,000. How long will it take her to first reach her goal? How much will she actually have in the account when she first reaches her goal?

11. Benjamin is 25 years old and plans to retire in 40 years. When he retires, he would like to receive payments of \$3,000 at the beginning of each month for 15 years from a retirement account.
 - (a) How much money should Benjamin deposit at the end of each month from now until he retires to achieve this goal if he secures an account that will pay 6.25%/year compounded monthly for the life of the account?
 - (b) How much will Benjamin deposit into this account?
 - (c) How much interest will be earned over the entire life of the account?
12. Smith borrows 1000 at an 8% annual effective interest rate and will pay it back with level annual payments over 30 years. How much more should Smith pay each year so that he can clear his debt in 20 years?
13. Seth pays 10,000 for an annuity-immediate that pays 400 quarterly for the next 10 years. Calculate the annual nominal interest rate convertible monthly earned by Seth's investment.
14. The accumulated value of an annuity-immediate that pays 21.80 annually for 10 years is equal to the present value of a perpetuity-immediate that pays 19.28 annually when valued at the same interest rate i . Both are equal to X . Find X .
15. Jamal is considering buying a car. If he buys the car outright, he would pay 15,000 now, and at the end of 5 years, he could sell it for 8,000. If Jamal leases the car for 5 years, he must pay 1,400 now and then make payments of X at the end of each year. Find X that makes these two options equivalent, assuming that the effective annual rate of interest is 10%.
16. Rick receives payments of X at the beginning of each year for n years. The present value of his annuity is 493. Alice receives payments of $3X$ at the beginning of each year for $2n$ years. The present value of her annuity is 2,748. If both accounts earn the same annual effective rate of interest $i > 0$, calculate v^n . Give an exact answer.
17. A professor wants to endow a scholarship that would yield an award of \$5,000 at the end of each six month period, continuing forever, with the first award to be made three years from today. How much money must be deposited into an account today to endow this scholarship if the account is guaranteed to earn interest forever at a nominal annual rate of 6% compounded semiannually?
18. A perpetuity-due is purchased for 1,100 on January 1, 2000. The level annual payments are 100 and the interest rate is i compounded annually. Immediately following the payment on January 1, 2014, the remaining future payments are sold at a yield rate of i . The proceeds are used to purchase a 10-year annuity whose first payment will be on January 1, 2018, where semiannual payments are made on January 1 and July 1. The interest rate for the annuity (starting on January 1, 2014) is $0.75i$ compounded semiannually. Find the payment amount.