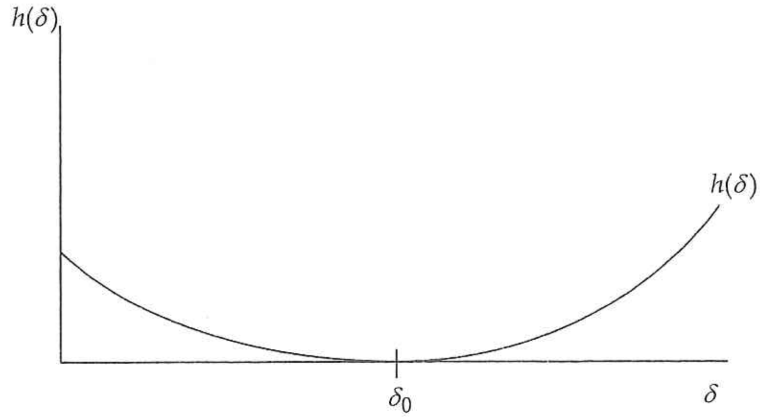

Since the minimum of $h(\delta)$ occurs at δ_0, the minimum of $h(\delta)$ is zero:

$$h(\delta_0) = e^{q(\delta_0-\delta_0)} + \frac{q}{r}e^{-r(\delta_0-\delta_0)} - \left(1+\frac{q}{r}\right) = 0$$

Therefore, for all values of δ other than δ_0, $h(\delta)$ is positive.

We have shown that both $g(\delta)$ and $h(\delta)$ are positive when $\delta \neq \delta_0$. Therefore, their product must also be positive when $\delta \neq \delta_0$. This means that:

$$S(\delta) > 0 \text{ for } \delta \neq \delta_0$$

Since the surplus is positive for any change in the interest rate from δ_0, the position is fully immunized.

7.9 Dedication

Dedication, which is also known as **cash flow matching**, calls for matching the asset and liability cash flows exactly. For each liability payment there is an equal asset payment made at the same time.

A dedicated portfolio of assets can be constructed to fund a set of known liabilities. Large pension plans, for example, are often able to predict their liability payments with a great deal of accuracy. Once the liability payments are known, the portfolio manager selects assets that provide cash flows to match the liability payments.

An advantage to dedication is that rebalancing is not necessary. A disadvantage of dedication is that it limits the universe of bonds that can be purchased. A bond might be attractively priced, but if its cash flows do not match up with the liability cash flows, then the portfolio manager is unable to buy the bond under a dedication strategy. Because the universe of bonds from which the portfolio manager can choose is limited, the portfolio's yield may not be as high as would be the case under immunization.

Example 7.16

A bank has an obligation to pay $1,000 in one year and $4,000 in 4 years. The bank has decided to pursue a dedication strategy. The annual effective yield on a 1-year zero coupon bond is 8%, and the annual effective yield on a 4-year zero coupon bond is 9%.

Calculate the cost of establishing the asset portfolio.

Solution

In order to match the liability cash flow of $1,000 in one year, the bank must purchase a zero-coupon bond that matures in one year for $1,000. The cost of this bond is $925.93:

$$\frac{1{,}000}{1.08} = 925.93$$

In order to match the liability cash flow of $4,000 in four years, the bank must purchase a zero-coupon bond that matures in four years for $4,000. The cost of this bond is $2,833.70:

$$\frac{4{,}000}{1.09^4} = 2{,}833.70$$

In total, the bank pays $3,759.63 to establish the asset portfolio:

$$925.93 + 2{,}833.70 = 3{,}759.63$$

♦♦

The preceding example illustrates that cash flow matching is a fairly simple process when zero-coupon bonds are used.

If bonds with positive cash flows are used, then the cash flow matching calls for matching the longest liability first and working backwards to the shortest liability. We begin by purchasing an asset that has a final cash flow that is equal to the final liability cash flow. The net liability cash flows remaining are those cash flows that are not offset by the asset cash flows. The new final net liability cash flow is identified and an another asset is then purchased to offset it. This process is continued until all of the liability cash flows are exactly offset by asset cash flows.

Example 7.17

A company has the following projected liability cash flows:

Year	1	2	3	4	5
Liability cash flow	179	679	144	3,144	824

There are three assets available for investment:

- 2-year bond with annual coupons of 7%
- 4-year bond with annual coupons of 4%
- 5-year bond with annual coupons of 3%

Each bond has a par value of $100. The annual effective yield on all three bonds is 5%.

The company has decided to pursue a dedication strategy. Determine the amount of each bond to be purchased, and calculate the cost of establishing the asset portfolio.

Solution

The final liability cash flow is $824. The final cash flow from the 5-year bond is its principal payment plus its coupon payment:

$$\text{Final cash flow of 5-year bond} = 100 + 3 = 103$$

We can determine the number of 5-year bonds that provides an asset cash flow of $824 in 5 years:

$$\text{Number of 5-year bonds to purchase} = \frac{824}{103} = 8.0$$

Subtracting the cash flows produced by the eight 5-year bonds from the liability cash flows gives us the net liability cash flows remaining:

Year	1	2	3	4	5
Liability cash flow	179	679	144	3,144	824
Cash flow from 8 5-year bonds	24	24	24	24	824
Net liability cash flow remaining	155	655	120	3,120	0

The new final net liability cash flow is \$3,120 at the end of year 4. This is offset by purchasing thirty of the 4-year bonds:

$$\text{Number of 4-year bonds to purchase} = \frac{3,120}{104} = 30.0$$

Subtracting the cash flows produced by the thirty 4-year bonds from the net liability cash flows gives us the new net liability cash flows remaining in the bottom row of the table below:

Year	1	2	3	4	5
Liability cash flow	179	679	144	3,144	824
Cash flow from 8 5-year bonds	24	24	24	24	824
Net liability cash flow remaining	155	655	120	3,120	0
Cash flow from 30 4-year bonds	120	120	120	3,120	0
Net liability cash flow remaining	35	535	0	0	0

The new final net liability cash flow is \$535 at the end of year 2. This is offset by purchasing five of the 2-year bonds:

$$\text{Number of 2-year bonds to purchase} = \frac{535}{107} = 5.0$$

Subtracting the cash flows produced by the five 2-year bonds from the net liability cash flows gives us the new net liability cash flows remaining in the bottom row of the table below:

Year	1	2	3	4	5
Liability cash flow	179	679	144	3,144	824
Cash flow from 8 5-year bonds	24	24	24	24	824
Net liability cash flow remaining	155	655	120	3,120	0
Cash flow from 30 4-year bonds	120	120	120	3,120	0
Net liability cash flow remaining	35	535	0	0	0
Cash flow from 5 2-year bonds	35	535	0	0	0
Net liability cash flow remaining	0	0	0	0	0

The net liability cash flows are now all zero, so the purchase of eight 5-year bonds, thirty 4-year bonds, and five 2-year bonds results in a cash-matched portfolio.

In order to determine the cost of the asset portfolio, we must determine the price of each asset:

$$\text{Price of 2-year bond} = 7a_{\overline{2}|5\%} + \frac{100}{1.05^2} = 103.7188$$

$$\text{Price of 4-year bond} = 4a_{\overline{4}|5\%} + \frac{100}{1.05^4} = 96.4540$$

$$\text{Price of 5-year bond} = 3a_{\overline{5}|5\%} + \frac{100}{1.05^5} = 91.3410$$

To find the cost of establishing the portfolio, we sum the cost of purchasing five 2-year bonds, 30 4-year bonds, and eight 5-year bonds:

$$\text{Cost to establish asset portfolio} = 5 \times 103.7188 + 30 \times 96.4540 + 8 \times 91.3410 = 4{,}142.94$$

The cost to establish the asset portfolio is $4,142.94. ♦♦