Math 166.200

Exam III

Tuesday, November 13, 2012

Printed Name: _____

Signature: Key

- **You must show all appropriate work to receive full credit.**
- If you give a decimal answer that is not money, then round to at least 4 decimal places.
- There are 100 points possible. Point values for each problem are as indicated.
- Be careful in converting percentages 2% = .02
- SCHOLASTIC DISHONESTY WILL NOT BE TOLERATED.
- If you need more space to work a problem, you may use the back of the exam. Please indicate where the problem is located.
- INFORMATION ABOUT CARDS. Each standard deck of cards has 52 cards. There are 4 suits in a deck of cards; Hearts, Diamonds, Spades, and Clubs. The Hearts and Diamonds are both red, while the Spades and Clubs are both black. Within each suit there are 13 cards and they are labeled Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King. If you feel that you need more information about a card deck please ask.
- There are 26 letters in the English alphabet.
- there are 33 letters in the Russian alphabet.
- Formula that was requested: $P_\lambda(X) = \frac{\lambda^x}{x!} * e^{-\lambda}$

Good Luck!

1. (5 points) Sara took out a discount loan. She received \$800 and has agreed to payback the loan in 8 months. What was the discount on this loan if the monthly discount rate is 2%?

N = I= PV= Pmt= FV= P/y=C/y=

$P = m(1-rt)$

$800 = m(1-.02(8))$

$m = 952.38$

$D = m - P$

$= 152.38$

Answer: 152.38

2. (5 points) You place \$600 into an account and in 1 year you have \$850. What monthly simple interest rate is needed for this to happen?

N = I= PV= Pmt= FV= P/y=C/y=

$I = Prt$

$250 = 600\, r\, (12)$

$r = 3.472\%$

Answer: 3.472%

3. (5 points) How much do you start an account with if you will need \$8,000 in 5 years if the account has an annual interest rate of 6.2% compounded semiannually?

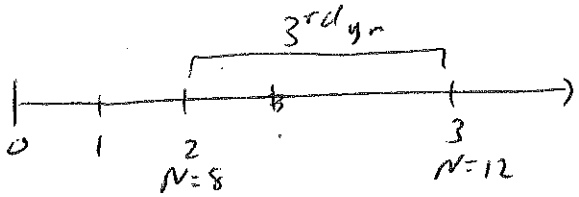
N = 5(2) I= 6.2 PV= ? Pmt= 0 FV= 8000 P/y=C/y= 2

Answer: ~~\$ 5721.19~~ 5895.27

4. (6 points) You start an account with \$500 and make payments of \$200 every quarter. If the interest rate of the account is 5% per year compounded quarterly, how much interest did you earn in the third year of the account?

N = I= 5 PV= -500 Pmt= -200 FV= ? P/y=C/y= 4

Answer: 128.43



Balance N=12 − Balance N=8 − any pmts

$3152.45 - 2224.02 - 4(200) = 128.43$

Check the back of the page for more problems.

5. (15 points) Collin is buying a house for \$85,000. He will finance the loan at 6% interest compounded monthly for the next 30 years.

   (a) What payment should Collin make in order to pay off the house?

N = 30(12) I= 6 PV= 85000 Pmt= ? FV= 0 P/y=C/y= 12

Answer: 509.62

   (b) How much equity would Collin in the house after 9 years?

N = 9(12) I= 6 PV= 85000 Pmt= -509.62 FV= ? P/y=C/y= 12

Answer: \$ 12,078.65

85000 − 72921.35

= \$12,078.65

   (c) Collin is looking into refinancing the loan. He can get a new loan for 15 years at a rate of 5% compounded monthly. There is a \$5000 fee to refinance the loan. Assume that this fee is added to the amount to be refinanced. Should he accept this refinancing offer if he has already paid off 9 years of the first mortgage? Justify your answer.

72921.35 + 5000

N = 15(12) I= 5 PV= 77921.35 Pmt= ? FV= 0 P/y=C/y= 12

new payment = 616.20

paid on new loan = 616.20(12)(15)
= \$110,916

left to pay on old loan = 509.62(12)(21) =
= 128,424.24

yes

will save \$17,508.24

Check the back of the page for more problems.

6. (5 points) Bob currently owes \$5000 to Stacy for a loan that he got 4 years ago. The loan has an interest rate of 8% compounded annually and Bob has been making yearly payments of \$800. How much money did Bob borrow from Stacy?

N = 4 I= 8 PV= ? Pmt= -800 FV= -5000 P/y=C/y= 1

Answer: \$6324.85

7. (4 points) A village has two industries: A and B. Production of one unit of A requires 0.2 units of A and 0.4 unit of B. Production of one unit of B requires 0.3 units of A and 0.1 units of B. Find the input-output matrix. Be sure to label the columns and rows.

$$\begin{array}{c} \\ A \\ B \end{array}\begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{bmatrix} .2 & .3 \\ .4 & .1 \end{bmatrix} \end{array}$$

8. (8 points) An economy has three sectors: raw rubber (R), tires(T), and other rubber goods(G). The input-output matrix A is shown below including the sector labels.

$$A = \begin{array}{c} \\ R \\ T \\ G \end{array}\begin{array}{c} \begin{array}{ccc} R & T & G \end{array} \\ \begin{bmatrix} 0.3 & 0.4 & 0.2 \\ 0.4 & 0.2 & 0.1 \\ 0.2 & 0 & 0.4 \end{bmatrix} \end{array} \qquad D = \begin{bmatrix} 150 \\ 60 \\ 80 \end{bmatrix} \qquad X = (I-A)^{-1} \cdot D$$

(a) There is a demand for 150 units of raw rubber, 60 units of tires, and 80 units of other rubber goods. Find the production level that would satisfy this demand.

R: 512 T: 369 G: 304

(b) How much of the production(in part a) is consumed internally?

R: 362 T: 309 G: 224

$$AX = X - D = \begin{bmatrix} 512 \\ 369 \\ 304 \end{bmatrix} - \begin{bmatrix} 150 \\ 60 \\ 80 \end{bmatrix}$$

Check the back of the page for more problems.

9. (6 points) The input-output matrix of a closed economy, i.e. isolated village, is given below. This economy has three sectors: farming, building, and services. If 10 units of service are being produce, when the economy is at equilibrium, how many units of farming are being produced?

$$A = \begin{array}{c} \\ \text{F} \\ \text{B} \\ \text{S} \end{array} \begin{bmatrix} \text{F} & \text{B} & \text{S} \\ 0.4 & 0.2 & 0.3 \\ 0.3 & 0.4 & 0.3 \\ 0.3 & 0.4 & 0.4 \end{bmatrix}$$

$$I - A = \begin{bmatrix} .6 & -.2 & -.3 \\ -.3 & .6 & -.3 \\ -.3 & -.4 & .6 \end{bmatrix}$$

$$\begin{bmatrix} .6 & -.2 & -.3 & 0 \\ -.3 & .6 & -.3 & 0 \\ -.3 & -.4 & .6 & 0 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & -.8 & 0 \\ 0 & 1 & -.9 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

F = .8S

B = .9S

S = any # ≥ 0

If S = 10 $\Rightarrow$ F = .8(10) = 8 units

10. (6 points) Compute the following. If it is not possible, then write not possible.

(a) $5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} b & 4 \\ -1 & 0 \\ 0 & a \end{bmatrix} =$ $\begin{bmatrix} 5b & 20 \\ -5 & 0 \\ 0 & 5A \end{bmatrix}$

(b) If $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 7 \\ 10 \end{bmatrix}$, compute $BA =$ not possible.

Check the back of the page for more problems.

11. (5 points) If $A = \begin{bmatrix} 2 & 1 \\ 7 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix}$, solve the equation $JA = B$ for J.

$J = BA^{-1} = \begin{bmatrix} 32 & -9 \\ 15 & -4 \end{bmatrix}$

12. (6 points) Solve for the variables in the problem. If it is not possible, then explain why.

$$\begin{bmatrix} 1 & 2y \\ x & 2 \end{bmatrix} - 4\begin{bmatrix} 3 & -2x+1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -11 & 22 \\ 10 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2y \\ x & 2 \end{bmatrix} - \begin{bmatrix} 12 & -8x+4 \\ -8 & 4 \end{bmatrix} = \begin{bmatrix} -11 & 22 \\ 10 & -2 \end{bmatrix}$$

$$\begin{bmatrix} -11 & 2y+8x-4 \\ x+8 & -2 \end{bmatrix} = \begin{bmatrix} -11 & 22 \\ 10 & -2 \end{bmatrix}$$

$x+8=10$

$x=2$

$2y+8x-4=22$

$2y+8x=26$

$2y+16=26$

$2y=10$

$y=5$

13. (8 points) Give the solution to each of these linear systems represented by the matrices. The variables are listed in the first row of the matrix.

$$\left[\begin{array}{ccc|c} x & y & z & \\ 5 & 0 & 0 & 30 \\ 0 & 2 & 0 & 10 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} x & y & z & \\ 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 0 & 3 \end{array}\right]$$

$\emptyset$

$x=6$

$y=5$

$z=4$

Check the back of the page for more problems.

14. (6 points) Set up the equation that represents this information. Be sure to define the variables as shown in class.

A box has bills in one-dollar, five-dollar and ten-dollar denominations. There are twice as many five-dollar bills as ten-dollar and one-dollar bills combined.

x = # of \$1 bills
y = # of \$5 bills
z = # of \$10 bills

$$y = 2(x + z)$$

15. (10 points) One hundred toys were given to a group of children. Based on the information in the problem, this problem has been worked to this point. Finish solving the problem placing restrictions on the parameter if necessary.

x = the number of dolls given.
y = the number of paintball guns given.
z = the number of slingshots given.

$$\left[\begin{array}{ccc|c} x & y & z & \\ 1 & 0 & -1 & 5 \\ 0 & 1 & 2 & 95 \end{array}\right]$$

$x = z + 5$

$y = 95 - 2z$

z = any #

$x \geq 0$

$z + 5 \geq 0$

$z \geq -5$

$y \geq 0$

$95 - 2z \geq 0$

$-2z \geq -95$

$z \leq 95/2 = 47.5$

$x \leq 100$

$x + 5 \leq 100$

$x \leq 95$

$y \leq 100$

$95 - 2z \leq 100$

$-2z \leq 5$

$z \geq -2.5$

Restriction

$z = 0, 1, 2, \ldots 47$