

**Week in Review # 8**

1.  $x$  = the number of senior tickets sold.  
 $y$  = the number of adult tickets sold.  
 $z$  = the number of children tickets sold.

$$\begin{aligned}x + y + z &= 700 \\6x + 8y + 3.5z &= 3512.5 \\3y &= z\end{aligned}$$

2.  $x$  = the number of Boeing 747s bought.  
 $y$  = the number of Boeing 777s bought.  
 $z$  = the number of Airbus A321s bought.

$$\begin{aligned}x + y + z &= 11 \\400x + 300y + 200z &= 3200 \\200x + 160y + 60z &= 1540\end{aligned}$$

3.  $x$  = the amount invested in low-risk stocks.  
 $y$  = the amount invested in high-risk stocks.  
 $z$  = the amount invested in bonds.

$$\begin{aligned}x + y + z &= 82000 \\y &= x + z \\0.08x + 0.15y + 0.04z &= 9050\end{aligned}$$

4. (a) no solution

(b)  $x = 9$ ,  $y = 10$ , and  $z = 6$

(c)  $x = 2 - 4z$   
 $y = 9 - 5z$   
 $z = \text{any number}$

(d)  $x = 7 - 2y - 2w$   
 $z = 3 - 4w$   
 $y = \text{any number}$   
 $w = \text{any number}$

(e)  $x = 4$ ,  $y = 2$ , and  $z = 8$

5. The row operations that need to be performed are:  $R_2 + 5R_1 \rightarrow R_2$  and  $R_3 + (-4)R_1 \rightarrow R_3$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 9 & 12 \\ 0 & 2 & 46 & 63 \\ 0 & 2 & -39 & -40 \end{array} \right]$$

6. 
$$\left[ \begin{array}{ccc|c} 3 & 0 & 23 & 17 \\ 7 & 11 & 39 & 25 \\ 10 & 0 & 1 & 16 \\ 0 & 5 & 6 & 1 \end{array} \right]$$

7. (a) first rewrite the equations as shown.

$$\begin{aligned}3x + y &= 9 \\x - y + z &= 4 \\3x + z &= 11 \\4x - y + 2z &= 15\end{aligned}$$

$$\left[ \begin{array}{ccc|c} 3 & 1 & 0 & 9 \\ 1 & -1 & 1 & 4 \\ 3 & 0 & 1 & 11 \\ 4 & -1 & 2 & 15 \end{array} \right] \xrightarrow{\text{rref}} \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Answer:  $x = 2$ ,  $y = 3$ , and  $z = 5$

(b) 
$$\left[ \begin{array}{ccc|c} 1 & 3 & 1 & 10 \\ 2 & 7 & -1 & 21 \\ 4 & 13 & 1 & 41 \end{array} \right] \xrightarrow{\text{rref}} \left[ \begin{array}{ccc|c} 1 & 0 & 10 & 7 \\ 0 & 1 & -3 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Answer:

$$x = 7 - 10z$$

$$y = 1 + 3z$$

$z = \text{any number}$ .

note: no restrictions can be placed on the parameter since this was not a word problem.

(c) 
$$\left[ \begin{array}{ccc|c} 3 & 2 & 5 & 7 \\ 1 & 4 & 1 & 13 \\ 4 & -5 & 2 & -9 \\ 5 & 10 & 7 & 32 \end{array} \right] \xrightarrow{\text{rref}} \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]$$

Answer: no solution.

8. (a) Set up of the problem:

$x$  = the number of old dvds bought

$y$  = the number of semi-new dvds bought

$z$  = the number of new dvds bought.

$$x + y + z = 60$$

$$10x + 16y + 22z = 840$$

Solution:

$$x = 20 + z$$

$$y = 40 - 2z$$

$z = \text{any number}$

Now place restrictions on the parameter  $z$ . This is the mathematical process. You could also do this by inspecting the parametric solution for what values of  $z$  will make sense.

We know that the number of dvds bought must be greater than or equal to zero.

$$\begin{array}{lll} x \geq 0 & y \geq 0 & z \geq 0 \\ 20 + z \geq 0 & 40 - 2z \geq 0 & \\ z \geq -20 & 40 \geq 2z & \\ & 20 \geq z & \end{array}$$

We also know that the number of dvds bought must be less than 60.

$$\begin{array}{lll} x \leq 60 & y \leq 60 & z \leq 60 \\ 20 + z \leq 60 & 40 - 2z \leq 60 & \\ z \leq 40 & -2z \leq 20 & \\ & z \geq -10 & \end{array}$$

Thus we get that  $0 \leq z \leq 20$  and  $z$  must be an integer or in other words  $z = 0, 1, 2, 3, \dots, 20$

(b) 21 different solutions.

9. (a)  $3d_{2,2} + 2c_{2,1} = 3(5) + 2(-2) = 11$

(b)  $\begin{bmatrix} 21 & 6 & 12 \\ 18 & 15 & 0 \end{bmatrix}$

(c)  $\begin{bmatrix} 1 & -2 & 2 \\ 3 & 5 & 0 \end{bmatrix}$

(d)  $\begin{bmatrix} 25 & 8 & 4 \\ 4 & 9 & 16 \end{bmatrix}$

(e)  $\begin{bmatrix} -15 & -6 & 12 \\ 22 & 7 & -32 \end{bmatrix}$

(f) not possible, wrong sizes.

(g)  $\begin{bmatrix} -9 & 3 \\ -14 & 15 \\ 2 & 0 \end{bmatrix}$

10. simplify the left and right side.

$$\begin{bmatrix} 19 & 8x-3y \\ 4y-18 & 10 \end{bmatrix} = \begin{bmatrix} 19 & -28 \\ x & 10 \end{bmatrix}$$

Now solve

$$8x - 3y = -28$$

$$4y - 18 = x$$

Answer:  $x = -2, y = 4$