Chapter 4 Homework Solutions
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1. (a) $\left[\begin{array}{ccc|c} 1 & 0 & 1 & -12 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & 7 & -7 \end{array}\right]$

   (b) $\left[\begin{array}{ccc|c} 1 & 0 & 9 & 12 \\ 0 & 2 & 19 & 27 \\ 0 & 2 & -12 & -4 \end{array}\right]$

   (c) $\left[\begin{array}{ccc|c} 1 & 2 & 5 & 3 \\ 0 & -2 & 4 & 4 \\ 0 & 18 & 3 & 10 \\ 3 & 0 & 6 & 1 \end{array}\right]$

   (d) $\left[\begin{array}{ccc|c} 4 & 0 & 26 & -7 \\ 0 & 4 & -2 & 5 \\ 0 & 7 & 74 & 35 \\ 5 & 1 & 2 & 5 \end{array}\right]$

2. (a) $x = 8$, $y = 2$, and $z = 5$

   (b) $x = 0$, $y = 4$, and $z = 2$

   (c) No solution.
   Note: no solution mean no solution for ALL of the variables. Do not say that x=8, z=5, no solution.

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3. x = the number of dimes
   y = the number of quarters

   $x + y = 32$
   $.1x + .25y = 5.15$

   Solution:
   19 dimes
   13 quarters

4. X = the number of nickels
   y = the number of quarters

   $x + y = 150$
   $.05x + .25y = 18.50$

   Solution:
   95 nickels
   55 quarters

5. x = amount invested in Fund A
   y = amount invested in Fund B

   $x + y = 50000$
   $.074x + .098y = 4072$

   Solution:
   $34,500 invested in Fund A
   $15,500 invested in Fund B

6. x = number of five dollar bills
   y = number of ten dollar bills
   z = number of twenty dollar bills

   $x + y + z = 70$
   $5x + 10y + 20z = 740$
   $x - 3y = 0$

   Solution:
   36 five dollar bills
   12 ten dollar bills
   22 twenty dollar bills

7. x = number of minutes jogging
   y = number of minutes playing handball
   z = number of minutes riding a bike

   $x + y + z = 60$
   $11x + 13y + 5z = 660$
   $x - 2z = 0$

   Solution:
   20 minutes jogging
   30 minutes playing handball
   10 minutes riding a bike

8. x = number of hours Valley Mills is scheduled.
   y = number of hours Marlin is scheduled.
   z = number of hours Hillsboro is scheduled.

   $10x + 7y + 5z = 1365$
   $12x + 10y + 4z = 1530$
   $6x + 8y + 13z = 1890$

   Solution:
   Valley Mills scheduled for 60 hours
   Marlin scheduled for 45 hours
   Hillsboro scheduled for 90 hours

9. x = the number of carton A
   y = the number of carton B
   z = the number of carton C

   $2x + 6y + 4z = 50$
   $5x + 8y + 6z = 78$
   $3x + 2y + 10z = 52$

   Solution:
   4 of carton A
   5 of carton B
   3 of carton C

10. x = number of plain hamburgers
    y = number of double cheeseburgers
    z = number of regular cheeseburgers

    $x + y + z = 86$
    $x + 2y + z = 100$
    $4y + 2z = 140$

    Solution:
    30 plain hamburgers
    14 double cheeseburgers
    42 regular cheeseburgers

11. x = number of one-bedroom units
    y = number of two-bedroom units
    z = number of three-bedroom units

    $x + y + z = 225$
    $y + z = 2x$
    $x = 3z$

Solution:
x=75
y = 125
z = 25

12. x = number of children at the show
y = number of students at the show
z = number of adults at the show

$x + y + z = 900$
$2x + 3y + 4z = 2800$
$2z = x + y$

Solution:
x=200
y=400
z=300

13. x = number of barrels of mix A
y = number of barrels of mix B
z = number of barrels of mix C
w = number of barrels of mix D

$30x + 30y + 30z + 60w = 900$
$50z + 75y + 25z + 25w = 750$
$30x + 20y + 20z + 50w = 700$

Initial matrix

$$\left[\begin{array}{cccc|c} 30 & 30 & 30 & 60 & 900 \\ 50 & 75 & 25 & 25 & 750 \\ 30 & 20 & 20 & 50 & 700 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 10 \\ 0 & 1 & 0 & -1 & -5 \\ 0 & 0 & 1 & 2 & 25 \end{array}\right]$$

final matrix

Parametric Solution:
$x = 10 - w$
$y = -5 + w$
$z = 25 - 2w$
$w = any\ number$

Now place restrictions on the parameter. Since we are not told a maximum number of barrels that can be bought assume there is no limit. We know the number of barrels bought has to be non-negative.

$x \geq 0$	$y \geq 0$	$z \geq 0$	$w \geq 0$
$10 - w \geq 0$	$-5 + w \geq 0$	$25 - 2w \geq 0$	
$10 \geq w$	$w \geq 5$	$25 \geq 2w$	
$w \leq 10$		$12.5 \geq w$	
		$w \leq 12.5$	

The restriction on the parameter is that w must be an integer and $5 \leq w \leq 10$ (i.e. w = 5, 6, 7, 8, 9, 10)

14. x = number of cars purchased with 6,000 gallon capacity
y = number of cars purchased with 8,000 gallon capacity
z = number of cars purchased with 18,000 gallon capacity

$x + y + z = 24$
$6000x + 8000y + 18000z = 250000$

Initial matrix and final matrix:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 24 \\ 6000 & 8000 & 18000 & 250000 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & -5 & -29 \\ 0 & 1 & 6 & 53 \end{array}\right]$$

Parametric solution:
$x = -29 + 5z$
$y = 53 - 6z$
$z = any\ number$

Once again we can not buy a part of a tank car. So z must be an integer. We also know that all of the variables must be greater than or equal to zero.

$x \geq 0$	$y \geq 0$	$z \geq 0$
$-29 + 5z \geq 0$	$53 - 6z \geq 0$	
$5z \geq 29$	$53 \geq 6z$	
$z \geq 5.8$	$53/6 \geq z$	
	$z \leq \frac{53}{6} \approx 8.8333$	

In addition we know that the variables can not be any larger than 24

$x \leq 24$	$y \leq 24$	$z \leq 24$
$-29 + 5z \leq 24$	$53 - 6z \leq 24$	
$5z \leq 53$	$29 \leq 6z$	
$z \leq 10.6$	$29/6 \leq z$	
	$z \geq \frac{29}{6} \approx 4.8333$	

Taken all together, we find that $z = 6, 7, 8$.

This problem ends up having only three solutions.

15. x = the number of chihuahuas bought
y = the number of cats bought
z = the number of dogs bought

$x + y + z = 14$
$4x + 7y + 16z = 116$

Solution:
$x = -6 + 3z$
$y = 20 - 4z$
$z = any\ number$

We can not buy a part of a pet. So z must be an integer. We also know that all of the variables must be greater than or equal to zero.

$x \geq 0$	$y \geq 0$	$z \geq 0$
$-6 + 3z \geq 0$	$20 - 4z \geq 0$	
$3z \geq 6$	$20 \geq 4z$	
$z \geq 2$	$5 \geq z$	

In addition we know that the variables can not be any larger than 14

$x \leq 14$	$y \leq 14$	$z \leq 14$
$-6 + 3z \leq 14$	$20 - 4z \leq 14$	
$3z \leq 20$	$6 \leq 4z$	
$z \leq \frac{20}{3} \approx 6.6667$	$1.5 \leq z$	

Taken all together, we find that $z = 2, 3, 4,$ or 5

16. s = the number of small shirts at the end of the day.
m = the number of medium shirts at the end of the day.
l = the number of large shirt at the end of the day.

$s + m + l = 45$
$8s + 10m + 13l = 480$
$8(2s) + 10(4m) + 13(5l) = 1940$

Solution:
15 small shirts
10 medium shirts
20 large shirts

17. x = number of tank cars purchased with 7,000 gallon capacity
y = number of tank cars purchased with 9,000 gallon capacity
z = number of tank cars purchased with 20,000 gallon capacity

$x + y + z = 40$
$7000x + 9000y + 20000z = 400000$

Solution:
$x = -20 + 5.5z$
$y = 60 - 6.5z$
$z = 4, 6,$ or 8

18. x = number of wood pens made
y = number of silver pens made
z = number of gold pens made

$x + .5y + 3z = 12000$
$2x + 3y + 2z = 9600$

Solution:
$x = 15600 - 4z$
$y = -7200 + 2z$
$3600 \leq z \leq 3900$ and z is an integer

19. x = the number of evil sorcerers slain.
y = the number of warriors slain.
z = the number of orcs slain.

$x + y + z = 370$
$2x + 4y + z = 560$
$y = 6x$

Solution:
10 evil sorcerers
60 warriors
300 orcs

20. x = the amount invested in the QX company
y = the amount invested in the RY company
z = the amount invested in the KZ company

$x + y + z = 17300$
$2z = y$
$1.5\left(\frac{x}{130}\right) + 1\left(\frac{y}{75}\right) + 2\left(\frac{z}{90}\right) = 251$

Solution:
\$6,500 invested in QX
\$7,200 invested in RY
\$3,600 invested in KZ

21. x = number of 12-ounce(small) cups sold
y = number of 16-ounce(medium) cups sold
z = number of 20-ounce(large) cups sold

$x + y + z = 23$
$12x + 16y + 20z = 376$
$x + 2y + 3z = 48$

Solution:
$x = z - 2$
$y = -2z + 25$
$z = 2, 3, 4, \cdots 12$

22. (a) the variables x, y, z, and w are the average number of vehicles on that section of the road.
Note: the number of vehicles entering the intersection must equal the number of vehicles exiting the intersection.
$x + y = 1400$
$y + z = 1200$
$z + w = 1100$
$x + w = 1300$
Solution:
$x = 1300 - w$
$y = 100 + w$
$z = 1100 - w$
w =any number

(b) $200 \leq w \leq 1000$

(c) To get this restriction we need $y = 150$. This means that $w = 50$. Since this is outside of the restrictions found in part b, the answer is no.

23. (a) $x + y = 1300$
$y + z = 1300$
$z + w - m = 800$
$v - m = 500$
$x + w - v = 300$
Solution:
columns in the matrix are x,y,z,v,m,w
$x = 800 + m - w$
$y = 500 - m + w$
$z = 800 + m - w$
$v = 500 + m$

(b) The only restriction that we can place on the parameters m and w is that they have to be non-negative, i.e. ≥ 0. We can not say anything else since the choice of m will affect the possible choices of w.

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24. (a) $x = 3 - 4y + w$
$z = 4 - 2w$
y, w = any number

(b) $x = 3 + z$
$y = 1 - 2z$
z = any number

(c) no solution

(d) $x = -2$, $y = 1$, $z = 3$

(e) no solution

25. (a) i. $x = 2 - 2z$
$y = 2 + z$
z = any number

ii. no solution

(b) i. $x = 12$, $y = -22$, and $z = 41$

ii. $x = \frac{-2}{17}$, $y = \frac{-10}{17}$, and $z = \frac{-60}{17}$

(c) no solution

(d) $x = 7 - 10z$
$y = 1 + 3z$
z = any number

(e) $x = 8$, $y = 3$, and $z = 5$

(f) $x = \frac{70}{3} + z$
$y = \frac{110}{3} - 2z$
z =any number

(g) $x = 1$, $y = -2$, $z = 3$, and $w = 8$

(h) $x = 26 - 2y + 14w$
$z = 7 + 6w$
y = any number
w = any number