Chapter 1 Homework Solutions
Compiled by Joe Kahlig

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1. (a) $\{0, 1, 11, 13\}$

   (b) $B \cap A = \{11\}$
   Answer: $\{0, 1, 11, 13\}$

   (c) $\{1, 13\}$

   (d) False
   It is true that $\emptyset \subset A$, but it is not an element of A.

   (e) False
   C contains 0 where as A does not.

2. (a) $A \cup B = \{0, 2, 3, 4, 5, 6, 7, 8\}$
   $(A \cup B)^C = \{1, 9\}$
   Answer: $\{1\}$

   (b) $\{2\}$

   (c) $C^C = \{1, 3, 4, 5, 6, 7, 8\}$
   $B \cap C^C = \{3, 4, 5, 6, 7, 8\}$
   Answer: $\{0, 2, 3, 4, 5, 6, 7, 8\}$

   (d) $D \cap A = \{2, 4, 8\}$
   $(D \cap A)^C = \{0, 1, 3, 5, 6, 7, 9\}$
   Answer: $\{3, 5, 6, 7\}$

   (e) $\{\}$ or $\emptyset$

   (f) $A \cup D = \{0, 1, 2, 3, 4, 5, 6, 8\}$
   Answer: $\{7, 9\}$

   (g) $D^C = \{0, 6, 7, 9\}$
   $B \cap D^C = \{6, 7\}$
   $(B \cap D^C)^C = \{0, 1, 2, 3, 4, 5, 8, 9\}$
   $A^C = \}1, 3, 5, 7, 9\}$
   Answer: $\{1, 3, 5, 9\}$

   (h) $D^C = \{0, 6, 7, 9\}$
   Answer: $\{6, 7\}$

   (i) U or $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

   (j) Yes

   (k) No , it is true that $\emptyset \subset A$, but it is not an element of A.

   (l) No, 4 is a number and not a set.

   (m) $2^5 = 32$

   (n) $2^6 - 1 = 63$

   (o) Yes

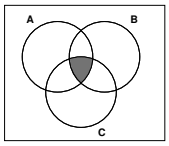
   (p) No

3. $\emptyset$, $\{m\}$, $\{n\}$, $\{p\}$, $\{m, n\}$, $\{m, p\}$, $\{n, p\}$, $\{m, n, p\}$

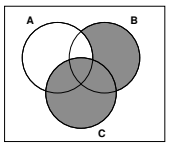
4. $\emptyset$, $\{m\}$, $\{n\}$, $\{p\}$, $\{m, n\}$, $\{m, p\}$, $\{n, p\}$

5. Remember one method is to label the regions of the venn diagram with letters and then shade the regions that are given in the answer to the computation of the set operation.

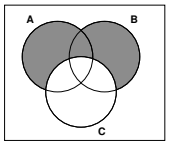
   (a) $A \cap B \cap C$

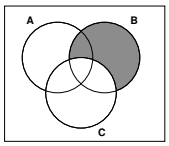
   (b) $(A^C \cap B) \cup C$

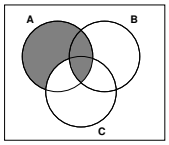
   (c) $(A \cup B) \cap C^C$

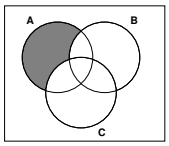
   (d) $B \cap C^C$

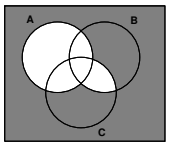
   (e) $A \cap (B \cup C^C)$

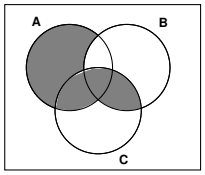
   (f) $A \cap B^C \cap C^C$

   (g) $A^C \cap (B^C \cup C^C)$

   (h) $(A \cap B^C) \cup (B \cap C)$

6. (a) $(A \cup M \cup E)^C$

   (b) $(E \cup M) \cap A^C$

   (c) Those students that have not had a course in Economics but have had a course in Accounting.

   (d) The students that have had a Marketing class but not an Accounting class combined with the students that have had an Economics class.

7. (a) $(D \cup C) \cap F^C$
   (b) $D \cap C^C \cap F^C$ or $D \cap (C \cup F)^C$

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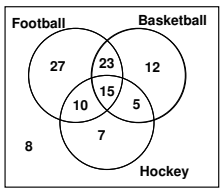
8. (a) $2^{14} = 16384$
   (b) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
   $18 = 14 + 10 - n(A \cap B)$
   $n(A \cap B) = 6$
   (c) $A \cap B^C$ is how many things are in A but not in B. Since A has 14 items and A and B have an overlap of 6 items then there are 8 items in A but not in B. Or use a venn diagram.
   Answer: 8

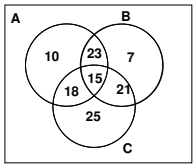
9. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.
   $n(A \cup B) = 15 + 12 - 7$
   $n(A \cup B) = 20$

10. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.
    $25 = 15 + n(B) - 7$
    $n(B) = 17$

11. venn diagram



12. It is not necessary to make a venn diagram, but it helps.



   (a) 119
   (b) $23 + 21 + 18 = 62$
   (c) 23
   (d) $25 + 21 = 46$

13. (a) $20 + 15 + 10 + 8 + 12 + 9 = 74$
   (b) 10
   (c) $10 + 15 + 9 + 12 + 20 + 45 + 60 = 171$
   (d) $15 + 12 = 27$

14. (a) $20 + 12 + 45 = 77$
   (b) $15 + 12 + 8 + 9 + 10 + 45 = 99$
   (c) $15 + 9 + 12 = 36$
   (d) $60 + 45 = 105$

15. (a) $30 = 8 + 10 + 12$
   (b) $62 = 8 + 12 + 8 + 12 + 12 + 10$
   (c) $28 = 8 + 12 + 8$

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16. (a) S = { (h,r), (h,w), (t,r), (t,w)}
   (b) there are multiple answers for this part.
   E = { (h,r), (h,w)}
   F = { (t,w)}

17. (a) Note: since we are drawing them out simultaneously, we don't care about the order. i.e. (1,2) is the same as (2,1)
   $S = \{(1,2),(1,3),(1,4),(1,5),(1,6),(1,7),(2,3),(2,4),(2,5),(2,6),(2,7),(3,4)(3,5),(3,6),(3,7),(4,5),(4,6),(4,7),(5,6),(5,7),(6,7)\}$
   (b) $E = \{(1,3),(1,5),(1,7),(3,5),(3,7),(5,7)\}$
   (c) $F = \{(2,4),(2,6),(4,6)\}$
   (d) no, missing the one even and one odd draws.
   (e) There are multiple answers for this part.
   G = {(1,3), (1,4), (1,5), (2,3)}
   H = {(2,5), (3,4), (3,5), (4,5)}

18. S = { 6, 10, 11, 15, 20}

19. Answers will vary.
   E = {HHH}
   F = {HHT, HTT, TTT}

20. let w = white ball, g=green ball, and y = yellow ball.
   (a) S= { ww, wg, wy, gw, gg, gy, yw, yq}
   (b) G = { wg, gw, gy, yg}
   (c) answer will vary. pick E such that $E \cap G = \phi$
   E={ww, wy}

21. (a) S = {R, E, P, S, N, T, A, I, V }
   (b) $2^{n(S)} = 2^9 = 512$
   (c) E = {E,A, I}

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22. (a) not equally likely since the chance of drawing a red ball is more likely than drawing a white ball.
   (b) see part a for the answer since uniform and equally likely mean the same thing.

23. (a) $\frac{20+7}{90} = \frac{27}{90}$
   (b) $\frac{21}{90}$

24. $\frac{25+30}{210} = \frac{55}{210}$

25. (a) $\frac{6}{11}$
   (b) $\frac{6+2}{11} = \frac{8}{11}$

26. (a) $\frac{41}{713}$
   (b) $\frac{55+41+52}{713} = \frac{181-33}{713} = \frac{148}{713}$
   (c) $\frac{171+199-41}{713} = \frac{329}{713}$

(d) $\frac{199+141}{713} = \frac{340}{713}$

27. (a) $\frac{85+35}{300}$
    (b) $\frac{85}{300}$
    (c) $\frac{58}{300}$
    (d) $\frac{170+26+154-12-138}{300} = \frac{200}{300}$

28. (a) $\frac{30+20+10+10}{1000} = \frac{70}{1000}$
    (b) $\frac{90+290-30}{1000} = \frac{350}{1000}$
    (c) $\frac{250+320+260}{1000} = \frac{830}{1000}$

29. (a) X = a 4 on either die and Y = sum of 5.

Red Die

Green Die	1	2	3	4	5	6
1				XY		
2			Y	X		
3		Y		X		
4	XY	X	X	X	X	X
5				X		
6				X		

Answer: $\frac{2}{36}$

(b) X = a 3 on either die and Y = sum of 4.

Red Die

Green Die	1	2	3	4	5	6
1			XY			
2		Y	X			
3	XY	X	X	X	X	X
4			X			
5			X			
6			X			

Answer: $\frac{12}{36}$

(c) X = a 6 on red die and Y = number less than 3 on the green.

Red Die

Green Die	1	2	3	4	5	6
1	Y	Y	Y	Y	Y	XY
2	Y	Y	Y	Y	Y	XY
3						X
4						X
5						X
6						X

Answer: $\frac{2}{36}$

30. X = a 4 on either die and Y = sum of 7

	1	2	3	4	5	6
1				X		Y
2				X	Y	
3				XY		
4	X	X	XY	X	X	X

Answer: $\frac{11}{24}$

31. (a) $0.2 = 1 - (.15 + .25 + .4)$
    (b) $0.4 = .15 + .25$

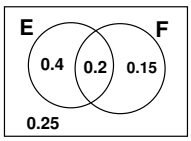
32. Since $P(a) + P(b) + P(c) = 1$ and $P(a) + P(b) = 0.75$, then $P(c) = 0.25$. Similarly $P(a) = 0.55$ and $P(b) = 0.2$.

33. $J^C = \{d, e\}$ which means that $P(d) + P(e) = 0.4$ and thus $P(d) = 0.25$. Since all probability adds up to 1 we get that $P(c) = 0.2$

34. $J^C = \{a, d, e\}$ which means that
    $P(a) + P(d) + P(e) = 0.45$
    $P(a) + 0.2 + 0.1 = 0.45$
    $P(a) = 0.15$

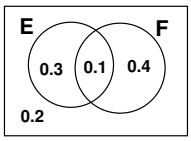
    Since a and b are equally likely, then $P(b) = 0.15$. Since all probability adds up to 1, we get that $P(c) = 0.4$

35. Use the information to fill in a venn diagram to answer part b and c.



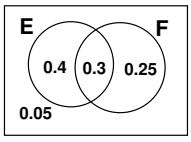
    (a) $0.4 + 0.2 = 0.6$ or $1 - 0.4 = 1 - P(E^C)$
    (b) 0.4
    (c) 0.8

36. Use the information to fill in a venn diagram to answer part c.



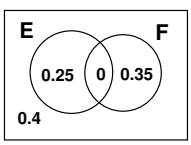
    (a) $0.4 = 1 - 0.6 = 1 - P(E^C)$
    (b) $0.1 = 0.4 + 0.5 - 0.8 = P(E) + P(F) - P(E \cup F)$
    (c) 0.3

37. Use the information to fill in a venn diagram to answer part b and c.



    (a) $0.55 = 1 - P(F^C)$
    (b) 0.3
    (c) $0.3 + 0.25 + 0.05 = 0.6$

38. $P(E \cap F) = 0$, since E and F are mutually exclusive. Use the information to fill in a venn diagram



(a) $P(E \cup F) = P(E) + P(F) - P(E \cap F)$
$P(E \cup F) = 0.25 + 0.35 - 0 = 0.6$

(b) 0.65

39. (a) $\frac{1}{6} + \frac{1}{8} + \frac{1}{8}$

(b) $\frac{1}{3} + \frac{1}{6}$

(c) $1 - \left(\frac{1}{3} + \frac{1}{6}\right)$

(d) 2^6. An event is the same as a subset.

(e) A and B are mutually exclusive
C and D are mutually exclusive

40. Use a venn diagram to organize the information.
Answer: $\frac{180+85}{500} = \frac{265}{500} = 0.53$

41. $\frac{7}{7+4} = \frac{7}{11}$

42. $\frac{23}{23+15} = \frac{23}{38}$

43. simplify $\frac{P(J)}{P(J^C)} = \frac{0.62}{0.38} = \frac{31}{19}$
Answer: 31 to 19

44. $P(A^C) = \frac{7}{15+7} = \frac{7}{22}$

45. $\frac{21}{40}$

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46. (a) $P(N|M) = \frac{P(N \cap M)}{P(M)} = \frac{0.25}{.4+.25} = \frac{0.25}{0.65}$

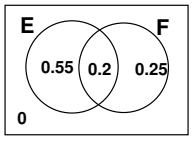
(b) $P(M|N) = \frac{P(M \cap N)}{P(N)} = \frac{0.25}{.15+.25} = \frac{0.25}{0.4}$

47. (a) $P(J|K) = \frac{P(J \cap K)}{P(K)} = \frac{.3}{.3+.22+.09} = \frac{.3}{.61}$

(b) $P(M|K^C) = \frac{P(M \cap K^C)}{P(K^C)} = \frac{.14}{.15+.14+.1} = \frac{.14}{.39}$

(c) $P(M|N) = \frac{P(M \cap N)}{P(N)} = \frac{0}{.09+.1} = 0$

48. Let E = solve the first problem and F = solve the second problem. Fill in a venn diagram with the given information.



(a) $P(F|E) = \frac{P(F \cap E)}{P(E)} = \frac{.2}{.75}$

(b) $P(E^C|F) = \frac{P(E^C \cap F)}{P(F)} = \frac{.25}{.45}$

49. First organize the information into a table.

	Fresh.(F)	Soph.(S)	Total
Male(M)	6	18	24
Female(F)	1	17	28
Total	7	36	42

Answer: $P(S|M) = \frac{18}{24}$

50. (a) $P(O|\text{only rifle}) = \frac{5}{26}$

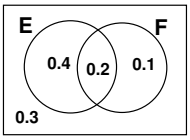
(b) $P(O|\text{own handgun}) = \frac{58+25}{120} = \frac{83}{120}$

(c) $P(F|\text{own rifle}) = \frac{12+5}{26+35} = \frac{17}{61}$

51. (a) $P(2\text{cds}|\text{over 25}) = \frac{40}{210}$

(b) $P(19 - -25|\text{lessthan2cds}) = \frac{70+110}{570} = \frac{180}{570}$

52. Use a venn diagram to organize the information.



(a) $\frac{P(F^C \cap E)}{P(E)} = \frac{0.4}{0.6}$

(b) $\frac{P(E^C \cap F^C)}{P(F^C)} = \frac{0.3}{0.7}$.

(c) $\frac{P(F \cap E^C)}{P(E^C)} = \frac{0.1}{0.4}$

53. (a) $P(A^C|C) = \frac{P(A^C \cap C)}{P(C)}$
$A^C \cap C = \{s_3, s_5\}$
Answer: $\frac{1/3+1/6}{1/8+1/3+1/6}$

(b) $P(C|B) = \frac{P(C \cap B)}{P(B)}$
$C \cap B = \{s_3, s_5\}$
Answer: $\frac{1/3+1/6}{1/3+1/6+1/12}$

54. (a) $0.6 * 0.3 + 0.4 * 0.2 = 0.26$

(b) $0.4 * 0.2 + 0.4 * 0.5 = 0.28$

(c) $0.6 * 0.7 + 0.4 * 0.3 = 0.54$

(d) $0.4 * 0.5 = 0.2$

(e) $P(A \cup G) = P(A) + P(G) - P(A \cap G)$
$P(A \cup G) = 0.6 + 0.54 - 0.42 = 0.72$

(f) 0.5

(g) 0.7

(h) $P(A|G) = \frac{P(A \cap G)}{P(G)} = \frac{0.6*0.7}{0.6*0.7+0.4*0.3}$

(i) $P(B|R) = \frac{B \cap R)}{P(R)} = \frac{0.4*0.5}{0.4*0.5} = 1$

(j) $P(A|Y) = \frac{A \cap Y)}{P(Y)} = \frac{0.6*0.3}{0.6*0.3+0.4*0.2}$

(k) $P(A|R) = \frac{A \cap R)}{P(R)} = \frac{0}{0.4*0.5} = 0$

55. (a) $0.1 * 0.2 + 0.6 * 0.7 = 0.44$

(b) $0.3 * 0.25 = 0.075$

(c) 0.8

(d) $P(C|G) = \frac{C \cap G)}{P(G)} = \frac{0.3*0.75}{0.6*00.3+00.3*0.75}$

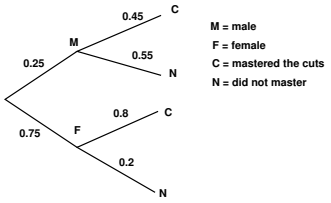
(e) $P(C) = 0.3$
$P(E) = 0.1 * 0.2 + 0.6 * 0.7 = 0.44$
$P(E \cap C) = 0$
Since $P(E \cap C) \neq P(E) * P(C)$ they are not independent.

(f) Yes since $P(E \cap C) = 0$

(g) $P(B) = 0.6$
$P(E) = 0.1 * 0.2 + 0.6 * 0.7 = 0.44$
$P(E \cap B) = 0.6 * 0.7 = 0.42$
$P(E) * P(B) = 0.6 * 0.44 = 0.264$
Since $P(E \cap B) \neq P(E) * P(B)$ they are not independent.

(h) No since $P(E \cap B) \neq 0$

56. Draw a tree.



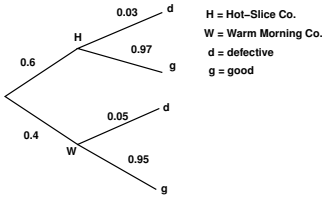
(a) $P(M|C) = \frac{P(M \cap C)}{P(C)} = \frac{0.25*0.45}{0.25*0.45+0.75*0.8}$

(b) $P(F \cup C) = .75 + .25 * .45 = 0.8625$

57. The third child has a good squirt gun so there are only 59 good guns remaining. Thus the second child culd pick any of the 20 bad squirt guns out of the total of 59+20=79 squirt guns.

Answer: $\frac{20}{79}$

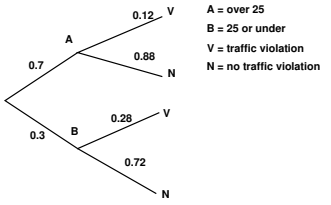
58. Draw a tree.



$P(d) = 0.6 * 0.97 + 0.4 * 0.95 = .962$

Answer: 96.2%

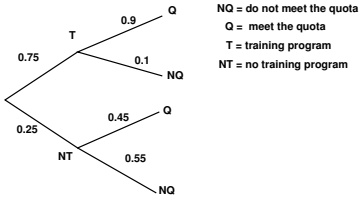
59. Draw a tree.



(a) $P(A|V) = \frac{P(A \cap V)}{P(V)} = \frac{0.7*0.12}{0.7*0.12+0.3*0.28}$

(b) $P(V) = 0.7 * 0.12 + 0.3 * 0.28 = 0.168$

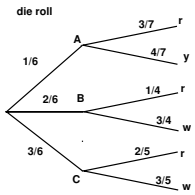
60. Draw a tree.



(a) $P(T|Q) = \frac{.75*.9}{.75*.9+.25*.45}$

(b) $P(NQ \cap NT) = .25 * .55$

61. Draw a tree.



(a) $P(C \cap W) = \frac{3}{6} * \frac{3}{5}$

(b) $P(B|r) = \frac{2/6*1/4}{1/6*3/7+2/6*1/4+3/6*2/5}$

62. A club and a diamond have been accounted for so there are still 13 hearts remaining and a total of 50 cards remaining.

Answer: $\frac{13}{50}$

63. (a) $\frac{12}{46}$

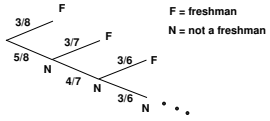
(b) $\frac{3}{46}$

(c) The seventh card was the king of hearts.
Answer: 0

64. prob of 5th card is a heart given the information is $\frac{12}{49}$

Answer: 12 to 37

65. Think of a tree.



you want $P(N \cap F)$.

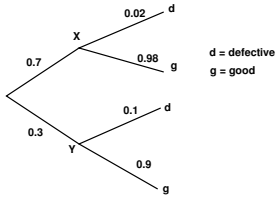
Answer: $\frac{5}{8} * \frac{3}{7} = \frac{15}{56}$

66. Draw a tree similar to the one from problem 65

(a) $\frac{4}{9} * \frac{3}{8} * \frac{5}{7} = \frac{60}{504}$

(b) By the fifth draw you have to have drawn a green ball. since you stop when you draw a green ball, you will never have a sixth draw.
Answer: 0

67. Draw a tree.

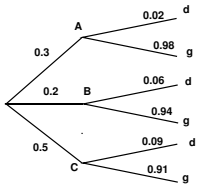


(a) $P(g) = 0.7 * 0.98 + 0.3 * 0.9 = 0.956$

(b) $P(Y|d) = \frac{0.3*0.1}{0.7*0.02+0.3*0.1}$

(c) $P(d \cap Y) = 0.1 * 0.3$

68. Draw a tree.



(a) $P(g \cap (B \cup C))P(g \cap B) + P(g \cap C) =$
$0.2 * 0.94 + 0.5 * 0.91 = 0.643$

(b) $P(g|C) = \frac{P(g \cap C)}{P(C)} = \frac{0.5*0.91}{0.5} = 0.91$

(c) $P(A|d) = \frac{P(A \cap d)}{P(d)} =$
$\frac{0.3*0.2}{0.3*0.02+0.2*0.06+0.5*0.09} = 0.095238$

69. (a) X = 3 or 4 on six sided die
Y = sum greater than 5.

	1	2	3	4	5	6
1			X	X	Y	Y
2			X	XY	Y	Y
3			XY	XY	Y	Y
4		Y	XY	XY	Y	Y

$P(X|Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{5/24}{14/24}$

Answer: $\frac{5}{14}$

(b) X= odd sum greater than 6
Y = 4 on either die

	1	2	3	4	5	6
1				Y		X
2				Y	X	
3				XY		X
4	Y		Y	XY	Y	XY

$P(X|Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{3/24}{9/24}$

Answer: $\frac{3}{9}$

(c) X= sum of 4
Y = sum at most 6

	1	2	3	4	5	6
1	Y	Y	XY	Y	Y	
2	Y	XY	Y	Y		
3	XY	Y	Y			
4	Y	Y				

$P(X|Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{3/24}{14/24}$

Answer: $\frac{3}{14}$

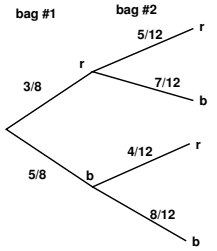
(d) X= sum of 4
Y = roll was a double

	1	2	3	4	5	6
1	Y		X			
2		XY				
3	X		Y			
4				Y		

$P(X|Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{1/24}{4/24}$

Answer: $\frac{1}{4}$

70. (a) probability tree.



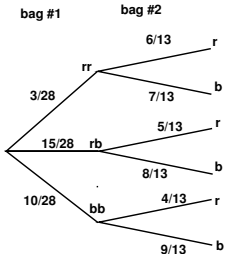
(b) $\frac{3}{8} * \frac{7}{12} + \frac{5}{8} * \frac{8}{12} = \frac{61}{96}$.

(c) P(2^{nd} r | 1^{st} b) $= \frac{4}{12}$

(d) P(1^{st} r | 2^{nd} b) $= \frac{21}{61}$

(e) P(1^{st} r | 2^{nd} 4) $= \frac{3}{7}$

71. (a) The probability of the first level of the tree was computed using combinations and then converting the answers to fractions.



(b) $\frac{15}{28} * \frac{8}{13} + \frac{10}{28} * \frac{4}{13} = \frac{40}{91}$

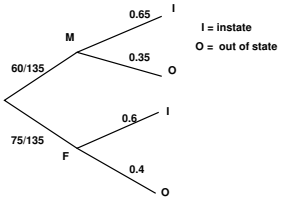
(c) $P(rr \cap b) = \frac{3}{28} * \frac{7}{13} = \frac{3}{52}$

(d) $P(r|rb) = \frac{5}{13}$

(e) $P(rb|r) = \frac{75}{133}$

(f) $P((rr \cup rb)|r) =$
$\frac{(3/28)*(6/13)+(15/28)*(5/13)}{(3/28)*(6/13)+(15/28)*(5/13)+(10/28)*(4/13)} = \frac{93}{133}$

72. Draw a tree.

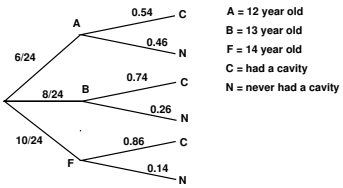


(a) $P(I|M) = 0.65$

(b) $P(F|O) = \frac{.4*75/135}{.35*60/135+.4*75/135} = \frac{10}{17}$

(c) $P(F) = \frac{75}{135} = \frac{5}{9}$
$P(O) = \frac{17}{45}$
$P(F \cap O) = \frac{2}{9}$
Since $P(F)P(O) = \frac{17}{81}$ is not equal to $P(F \cap O)$ these events are dependent. (i.e. not independent)

73. Draw a tree.



(a) $P(F|N) = \frac{.14*10/24}{.46*6/24+.26*8/24+.14*10/24} = \frac{35}{156}$

(b) $P(B) = \frac{8}{24}$
$P(C) = \frac{6}{24} * 0.54 + \frac{8}{24} * 0.74 + \frac{10}{24} * 0.86 = 0.74$
$P(B \cap C) = \frac{8}{24} * 0.74 = \frac{37}{150}$
$P(B) * P(C) = \frac{8}{24} * 0.74 = \frac{37}{150}$

Yes, since $P(B \cap C) = P(B) * P(C)$.

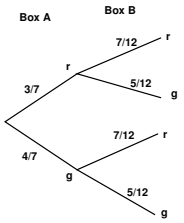
74. (a) Since E and F are independent then
$P(E \cap F) = P(E) * P(F)$
$P(E \cap F) = 0.6 * 0.3 = 0.18$

(b) $P(E \cup F) = P(E) + P(F) - P(E \cap F)$
$P(E \cup F) = 0.6 + 0.3 - 0.18$
Answer: 0.72

75. $P(E) = \frac{2}{9}$ and $P(F) = \frac{10}{29}$. Since E and F are independent, $P(E \cap F) = P(E) * P(F)$

$P(E \cap F) = \frac{2}{9} * \frac{10}{29} = \frac{20}{261}$

76. Since you are drawing an item from each box you can draw this tree to represent the problem.



Answer: $\frac{3}{7} * \frac{7}{13} + \frac{4}{7} * \frac{5}{13}$

77. Note the machines working or not working are independent.

(a) (A breaks down)*(B works all day) + (A works all day)*(B breaks down)
Answer: $0.02 * 0.97 + 0.98 * 0.03$

(b) (A works all day)*(B works all day)
Answer: $0.98 * 0.97$

78. $P(E) = \frac{2}{4}$
$P(F) = \frac{2}{4}$
$P(E \cap F) = \frac{1}{4}$
Since $P(E) * P(F) = \frac{2}{4} * \frac{2}{4} = \frac{1}{4} = P(E \cap F)$, these events are independent.

79. Similar to problem 78

Answer: Independent.

80. Similar to Problem 76

$0.075 * 0.87 + 0.925 * 0.13$

81. (a) $\frac{9}{10} * \frac{17}{20} * \frac{7}{15}$

(b) $\frac{1}{10} * \frac{17}{20} * \frac{7}{15} + \frac{9}{10} * \frac{3}{20} * \frac{7}{15} + \frac{9}{10} * \frac{17}{20} * \frac{8}{15}$