Applied Finite Mathematics
Tomastik / Epstein

G.1 Exercises

In Exercises 1 through 12, determine if any of the matrices are payoff matrices for strictly determined games. If one is, then find the value of the game and the optimal strategies for each player.

1. $\begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$

2. $\begin{bmatrix} -1 & 0 \\ -2 & -3 \end{bmatrix}$

3. $\begin{bmatrix} -3 & -2 & -4 \\ 0 & -2 & -1 \end{bmatrix}$

4. $\begin{bmatrix} 3 & 2 & 4 \\ 0 & 1 & 1 \end{bmatrix}$

5. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

6. $\begin{bmatrix} -1 & -2 \\ -3 & -4 \\ -5 & -6 \end{bmatrix}$

7. $\begin{bmatrix} -3 & -2 & -4 \\ 0 & -1 & -1 \end{bmatrix}$

8. $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 1 & 2 \end{bmatrix}$

9. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

10. $\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$

11. $\begin{bmatrix} 2 & -1 & -5 & -2 \\ 4 & -1 & 8 & 7 \\ 3 & 0 & 1 & 4 \end{bmatrix}$

12. $\begin{bmatrix} 4 & -4 & -2 & 0 \\ 0 & 0 & -1 & 0 \\ 9 & 3 & -3 & -2 \end{bmatrix}$

13. Show that the game with the payoff matrix

$$\begin{bmatrix} 0 & a \\ 1 & 2 \end{bmatrix}$$

is strictly determined no matter what a is.

14. Show that the game with the payoff matrix

$$\begin{bmatrix} 2 & 4 \\ 1 & b \end{bmatrix}$$

is strictly determined no matter what b is.

In Exercises 15 through 18, write the payoff matrix for the given game and determine if the game is strictly determined. If it is, find the optimal strategies for each player.

15. Each of two players R and C has, in their respective pockets, 3 coins: a penny, a nickel, and a dime. Each one selects a coin, and simultaneously each lays a coin on the table. If the coins are the same, no payment is made. If the coins are different, the one who played the coin with the smallest denomination wins both coins.

16. In this version of three-finger Morra, each player simultaneously shows either 1, 2, or 3 fingers. Player C agrees to pay player R an amount of dollars equal to the number of fingers shown by player R less the number shown by player C.

17. A person secretly places a penny, nickel, or dime in his fist. If you guess the correct coin, you win the coin. If you guess incorrectly, you give him the difference between your guess and the coin held.

18. In this version of four-finger Morra, each player simultaneously shows either 1, 2, 3, or 4 fingers. If the sum of fingers shown is even, you win an amount in dollars equal to the sum. If the sum of fingers is odd, you lose an amount equal to the sum.

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G.2 Exercises

In Exercises 1 through 4, find the expected return for the given mixed strategies, using the game with the indicated payoff matrix

$$\begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$$

1. $p = \begin{bmatrix} 0.50 & 0.50 \end{bmatrix}, q^T = \begin{bmatrix} 0.25 & 0.75 \end{bmatrix}$

2. $p = \begin{bmatrix} 0.50 & 0.50 \end{bmatrix}, q^T = \begin{bmatrix} 0.50 & 0.50 \end{bmatrix}$

3. $p = \begin{bmatrix} 0.30 & 0.70 \end{bmatrix}, q^T = \begin{bmatrix} 0.80 & 0.20 \end{bmatrix}$

4. $p = \begin{bmatrix} 0.60 & 0.40 \end{bmatrix}, q^T = \begin{bmatrix} 0.30 & 0.70 \end{bmatrix}$

In Exercises 5 through 8, find the expected return for the given mixed strategies, using the game with the indicated payoff matrix

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 3 & 4 \\ 2 & -1 & 3 \end{bmatrix}$$

5. $p = \begin{bmatrix} 0.50 & 0.50 & 0 \end{bmatrix}, q^T = \begin{bmatrix} 0.20 & 0 & 0.80 \end{bmatrix}$

6. $p = \begin{bmatrix} 0.10 & 0.20 & 0.70 \end{bmatrix}, q^T = \begin{bmatrix} 0 & 0.50 & 0.50 \end{bmatrix}$

7. $p = \begin{bmatrix} 0.30 & 0.30 & 0.40 \end{bmatrix}, q^T = \begin{bmatrix} 0.40 & 0.20 & 0.40 \end{bmatrix}$

8. $p = \begin{bmatrix} 0.30 & 0.60 & 0.10 \end{bmatrix}, q^T = \begin{bmatrix} 0 & 0.70 & 0.30 \end{bmatrix}$

Exercises 9 through 18 are payoff matrices. Find the value and the optimal strategies [redacted] Use dominance to eliminate any rows or columns.

9. $\begin{bmatrix} 1 & -2 \\ -1 & 2 \end{bmatrix}$ **10.** $\begin{bmatrix} 0 & -1 \\ -2 & 1 \end{bmatrix}$

11. $\begin{bmatrix} 0 & 2 \\ 1 & -1 \end{bmatrix}$ **12.** $\begin{bmatrix} -1 & 0 \\ 1 & -2 \end{bmatrix}$

13. $\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \end{bmatrix}$ **14.** $\begin{bmatrix} 1 & 1 & -1 \\ -1 & -2 & 1 \end{bmatrix}$

15. $\begin{bmatrix} -1 & 1 \\ 2 & -1 \\ 2 & -1 \end{bmatrix}$ **16.** $\begin{bmatrix} -2 & 0 \\ 1 & 0 \\ 1 & -1 \end{bmatrix}$

17. $\begin{bmatrix} 2 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & -1 & 2 \end{bmatrix}$ **18.** $\begin{bmatrix} 0 & -1 & 1 \\ 4 & 3 & -2 \\ 3 & 3 & 0 \end{bmatrix}$

The geometric theory given in the text can be used to find the optimal strategy for Row for any game with a payoff matrix with two rows. Find the optimal strategy for Row in Exercises 19 and 20.

19. $\begin{bmatrix} 3 & 0 & -1 \\ -5 & -4 & 2 \end{bmatrix}$ **20.** $\begin{bmatrix} 3 & 2 & -1 \\ -2 & 0 & 1 \end{bmatrix}$

The geometric theory given in the text can be used to find the optimal strategy for Column in any game with a payoff matrix with two columns. Find the optimal strategy for Column in Exercises 21 and 22.

21. $\begin{bmatrix} 4 & -2 \\ 1 & 2 \\ 3 & 1 \end{bmatrix}$ **22.** $\begin{bmatrix} 3 & -5 \\ -2 & 0 \\ -1 & -3 \end{bmatrix}$

Applications

23. **Setting Prices** Every August and January two carpet retailers in town run their sales. They must decide whether to have a 20% or 30% sale. From many previous years of experience with these sales, they know that if both set prices at 20% off the first retailer will get 80% of the business, and if both set prices at 30% off the first retailer will get 70% of the business. If the first retailer takes 20% off and the second 30% off, then the first retailer gets 30% of the business, while if the first takes 30% off and the second 20% off, the first retailer gets 60% of the business. What should their strategies be and what is the value of this game?

24. **Price Wars** Every day two movie theaters in town must decide on the price to set on their movies for the next day so the advertisement can go into the morning paper for the next day. They know from long experience that if they both set their prices at \$5 or both at \$6, then the first theater will get 70% of the business. If the first theater sets prices at \$5 and the second at \$6, then the first theater gets 60% of the business, while if the first theater charges \$6 and the second \$5, then the first theater gets 40% of the business. What should their strategies be and what is the value of this game?

25. **Advertising** Two competing electronics stores must decide each week whether to advertise in one and

only one of the three media: TV, radio, or newspaper. From past experience they know that the payoff matrix in terms of percent of the market gained or lost to the other is

$$\begin{array}{r} \\ \text{TV} \\ \text{Radio} \\ \text{Paper} \end{array}\begin{array}{c} \begin{array}{ccc} \text{TV} & \text{Radio} & \text{Paper} \end{array} \\ \left[\begin{array}{ccc} 1 & 1 & 0 \\ -1 & 0 & -1 \\ 2 & -1 & 2 \end{array}\right] \end{array}$$

Find the value of this game and the optimal strategies.

26. **Marketing** Early in each year two local growers compete for the freshly cut gladiola market, committing themselves entirely to one and only one color of gladiola. Past experience indicates the following payoff matrix given in terms of the percentage of the market the first grower gains or loses to the second.

$$\begin{array}{r} \\ \text{Pink} \\ \text{Yellow} \\ \text{Red} \end{array}\begin{array}{c} \begin{array}{ccc} \text{Pink} & \text{Yellow} & \text{Red} \end{array} \\ \left[\begin{array}{ccc} 20 & 10 & -20 \\ -10 & -20 & -10 \\ 30 & 20 & -20 \end{array}\right] \end{array}$$

Find the value of this game and the optimal strategies.

Extensions

27. Show the payoff matrix

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

has the value

$$v = \bar{p}A\bar{q} = \frac{a_{11}a_{22} - a_{12}a_{21}}{a_{11} + a_{22} - a_{12} - a_{21}}$$

when the row and column players use their optimum strategies as listed on page 373.

28. Show the optimal row and column strategies for the payoff matrix

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

are given by

$$p_1 = \frac{a_{22} - a_{21}}{a_{11} + a_{22} - a_{12} - a_{21}} \qquad p_2 = 1 - p_1$$

and

$$q_1 = \frac{a_{22} - a_{12}}{a_{11} + a_{22} - a_{12} - a_{21}} \qquad q_2 = 1 - q_1$$

Hint: Find the optimal strategies as in Examples 2 and 3 but use the given matrix for A. Rather than graphing the expected values, find the intersection algebraically.

Answers

G.1 EXERCISES

1. Value is 2. R picks second row. C picks first column.
3. Value is -2. R picks second row. C picks second column.
5. Value is 5. R picks third row. C picks first column.
7. Value is -1. R picks third row. C picks second column.
9. No
11. Value is 0. R picks third row. C picks second column.
13. No matter what a is, 1 is a saddle point.
15.

$$\begin{array}{cc} & \text{Player C} \\ & \begin{array}{ccc} \text{p} & \text{n} & \text{d} \end{array} \\ \text{Player R} \begin{array}{c} \text{p} \\ \text{n} \\ \text{d} \end{array} & \begin{bmatrix} 0 & 5 & 10 \\ -5 & 0 & 10 \\ -10 & -10 & 0 \end{bmatrix} \end{array}$$

Yes with value 0. Each player plays a penny.

17.

$$\begin{array}{cc} & \text{Opponent} \\ & \begin{array}{ccc} \text{p} & \text{n} & \text{d} \end{array} \\ \text{Row} \begin{array}{c} \text{p} \\ \text{n} \\ \text{d} \end{array} & \begin{bmatrix} 1 & -4 & -9 \\ -4 & 5 & -5 \\ -9 & -5 & 10 \end{bmatrix} \end{array}.$$

No

G.2 EXERCISES

1. 0.62 3. -0.52 5. 2.1 7. 1.56

9. $v = 0, p = \begin{bmatrix} 0.50 & 0.50 \end{bmatrix}, q^T = \begin{bmatrix} 2/3 & 1/3 \end{bmatrix}$

11. $v = 0.50, p = \begin{bmatrix} 0.50 & 0.50 \end{bmatrix}, q^T = \begin{bmatrix} 0.75 & 0.25 \end{bmatrix}$

13. $v = 0.50, p = \begin{bmatrix} 0.75 & 0.25 \end{bmatrix}, q^T = \begin{bmatrix} 0.50 & 0 & 0.50 \end{bmatrix}$

15. $v = -0.25, p = \begin{bmatrix} 0.75 & 0.25 & 0 \end{bmatrix}, q^T = \begin{bmatrix} 0.25 & 0.75 \end{bmatrix}$

17. $v = 0.50, p = \begin{bmatrix} 0 & 0.75 & 0.25 \end{bmatrix}, q^T = \begin{bmatrix} 0 & 0.50 & 0.50 \end{bmatrix}$

19. $v = -4/7, p = \begin{bmatrix} 6/7 & 1/7 \end{bmatrix}$

21. $v = 5/3, q^T = \begin{bmatrix} 1/3 & 2/3 \end{bmatrix}$

23. $v = 19/30$. The first should have a 20% sale $1/6$ of the time and have a 30% sale $5/6$ of the time. The second should have a 20% sale $2/3$ of the time and have a 30% sale $1/3$ of the time.

25. $v = 0.50$. The second should advertise one-half the time on radio and one-half the time in the newspaper. The first should advertise three-quarters of the time on TV and one-quarter of the time in the newspaper.

27. $v = \bar{p}A\bar{q}$

$$= \begin{bmatrix} \dfrac{a_{22} - a_{21}}{a_{11} + a_{22} - a_{12} - a_{21}} & 1 - \dfrac{a_{22} - a_{21}}{a_{11} + a_{22} - a_{12} - a_{21}} \end{bmatrix}$$