G1 EXERCISES Finite Mathematics by Rolf.

## Level 1

1. *(See Example 1)* Write the payoff matrix for the coin-matching game between two players. If both coins are heads, $R$ pays $C$ \$1. If both coins are tails, $C$ pays $R$ \$2. If the coins differ, $R$ pays $C$ 50¢.

2. Two players, $R$ and $C$, each have two cards. $R$ has one black card with the number 5 written on it and one red card with the number 3. $C$ has a black card with a 4 written on it and a red card with a 2. They each select one of their cards and simultaneously show the cards. If the cards are the same color, $R$ gets, in dollars, the difference of the two numbers shown. If the cards are different colors, $C$ gets, in dollars, the smaller of the two numbers shown. Write the payoff matrix of this game.

3. Two-Finger Morra is a game in which two players each hold up one or two fingers. The payoff, in dollars, is the total number of fingers shown. $R$ receives the payoff if the total is even, and $C$ receives the payoff if the total is odd. Write the payoff matrix.

4. Player $R$ has a \$1 bill, a \$10 bill, and a \$50 bill. Player $C$ has a \$5 bill and a \$20 bill. They simultaneously select one of their bills at random. The one with the larger bill collects the bill shown by the other. Write the payoff matrix for this game.

5. *(See Example 2)* Given the following payoff matrix:

$$\begin{array}{cc} & \begin{array}{ccc} & C & \\ \hline c_1 & c_2 & c_3 \end{array} \\ \begin{array}{c} \\ R \end{array} & \begin{array}{c} r_1 \\ r_2 \\ r_3 \end{array} \begin{bmatrix} 5 & -3 & 4 \\ 0 & 10 & -7 \\ -8 & 4 & 0 \end{bmatrix} \end{array}$$

**(a)** What is the payoff when strategies $r_2$ and $c_2$ are selected?
**(b)** What is the largest gain possible for the row player?
**(c)** What is the greatest loss possible for the column player?
**(d)** What is the largest gain possible for the column player?

**Exercises 6 through 11 are payoff matrices for two-person games. Decide whether the games are strictly determined. Find the saddle point, value, and solution for each strictly determined game.**

6. *(See Example 3)* $\begin{bmatrix} 5 & 2 \\ -4 & 1 \end{bmatrix}$

7. $\begin{bmatrix} -3 & -5 \\ 2 & -1 \end{bmatrix}$

8. $\begin{bmatrix} 5 & 9 \\ 7 & 0 \end{bmatrix}$

9. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & -5 & 1 \\ 2 & 6 & 3 \end{bmatrix}$

10. $\begin{bmatrix} -4 & 3 & 0 \\ -1 & 2 & 5 \\ -3 & 4 & -2 \end{bmatrix}$

11. $\begin{bmatrix} 2 & 0 & 1 \\ 0 & -3 & 4 \\ 3 & -2 & 0 \end{bmatrix}$

12. *(See Example 4)* Find the saddle points and values of the payoff matrices.

**(a)** $\begin{bmatrix} 3 & 3 & 8 \\ 1 & -2 & -3 \\ 3 & 3 & 9 \end{bmatrix}$ **(b)** $\begin{bmatrix} 6 & 7 & 6 & 8 \\ 3 & 6 & 5 & 12 \\ 6 & 9 & 6 & 11 \end{bmatrix}$

13. *(See Example 5)* For the following payoff matrices, find the saddle points, values, and solutions, if they exist.

**(a)** $\begin{bmatrix} -3 & 2 & 1 \\ 0 & 2 & 3 \\ -1 & -4 & 2 \end{bmatrix}$ **(b)** $\begin{bmatrix} -1 & 2 & 3 \\ 4 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$

**(c)** $\begin{bmatrix} 1 & -2 & 1 \\ 5 & 7 & 3 \\ -1 & 3 & -4 \end{bmatrix}$ **(d)** $\begin{bmatrix} -1 & -2 & 3 \\ 5 & 0 & 2 \\ 4 & -1 & 1 \end{bmatrix}$

## Level 2

**14.** Two coffeehouses near the campus, the Coffee Club and the Rendezvous Room, compete for the same customers. To attract more customers, the Coffee Club is considering adding a blues musical group or a cheesecake counter. There is room to add only one of these. At the same time, the Rendezvous Room is considering adding a jazz musical group or a bagel counter. They too have room to add only one.

The payoff matrix shows the percentage increase in business for the Rendezvous Room for each option adopted by the coffeehouses. Their options are no change, add music, and add food.

| | | Coffee Club | | |
|---|---|---|---|---|
| | | No Change | Add Blues | Add Cheesecake |
| Rendezvous Room | No Change | 0 | −10 | −15 |
| | Add Jazz | 15 | 2 | 5 |
| | Add Bagels | 10 | −5 | 6 |

Which option should each adopt? Find the resulting value.

**15.** Two television networks, Century and ReMark, are competing for a viewer audience in prime time. Century is considering a new talk show, a game show, and an educational documentary show. ReMark is considering a sports highlights show, a mystery drama, and a variety show. The following payoff matrix shows the estimated gain by ReMark in audience ratings for the pairings of shows.

| | | Century | | |
|---|---|---|---|---|
| | | Talk | Game | Educational |
| ReMark | Sports | 10 | −5 | 25 |
| | Mystery | −20 | −15 | 10 |
| | Variety | −10 | −10 | 5 |

Find the strategy each network should adopt and the value.

**16.** Crofford College and Round Rock Tech are neighboring schools that are considering tuition changes. Some argue that tuition should be increased to obtain more revenue, and others argue that tuition should remain the same or even be lowered to recruit more students. The decisions will cause some students to change from one school to the other. The following matrix describes student movement, with a positive entry indicating the number of students moving from Crofford to Round Rock and a negative number indicating movement from Round Rock to Crofford.

Determine if there is a saddle point. If so, what strategy should each school adopt?

| | | Crofford | | |
|---|---|---|---|---|
| | | Lower | Same | Raise |
| Round Rock | Lower | 125 | 200 | 320 |
| | Same | −175 | 80 | 175 |
| | Raise | −400 | −220 | 0 |

## Level 3

**17.** The farming occupation may be considered a game between the farmer, who has a choice of crops to plant, and nature, which has a "choice" of types of weather. For the type of soil and climate in his area, a farmer has the choice of planting crops of milo, corn, or wheat. The matrix shows the expected gross income per acre for each crop and weather condition for the growing season.

| | | Nature | | |
|---|---|---|---|---|
| | | Dry Season | Normal Season | Wet Season |
| Farmer | Milo | 85 | 120 | 150 |
| | Corn | 60 | 165 | 235 |
| | Wheat | 70 | 150 | 175 |

**(a)** Based on this "two-person" game, which crop strategy should the farmer adopt?

**(b)** The strategy of a two-person game assumes that each player is an intelligent person using a rational approach to the game. Do you think that game theory is a valid approach to determine which crop strategy is best?

**(c)** Weather records show that dry, normal, and wet seasons do not occur with equal frequency. Dry and wet seasons each occur about 20% of the time, and normal seasons occur about 60% of the time. For each crop, find the income over a five-year period assuming one year is dry, one is wet, and three are normal. Based on your findings, which is the best crop strategy for the long term? Is it the same as that found in part (a)?

# EXERCISES

Finite Mathematics by Rolf

## Level 1

**1.** *(See Example 1)* Find the expected payoff for the payoff matrix

$$\begin{matrix} & C \\ R & \begin{bmatrix} 20 & 12 \\ 8 & 30 \end{bmatrix} \end{matrix}$$

using $P = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \end{bmatrix}$ for $R$'s strategy and using $Q = \begin{bmatrix} \frac{1}{4} & \frac{3}{4} \end{bmatrix}$ for $C$'s strategy.

**2.** Find the expected payoff for the payoff matrix

$$A = \begin{bmatrix} 3 & -1 \\ -2 & 4 \end{bmatrix}$$

using the row strategy [0.3 0.7] and the column strategy [0.6 0.4].

**3.** Find the expected payoff for the payoff matrix

$$A = \begin{bmatrix} 3 & -6 \\ -2 & 4 \end{bmatrix}$$

using the row strategy $\begin{bmatrix} \frac{2}{5} & \frac{3}{5} \end{bmatrix}$ and the column strategy $\begin{bmatrix} \frac{2}{3} & \frac{1}{3} \end{bmatrix}$.

**4.** Find the expected payoff for the payoff matrix

$$\begin{matrix} & C \\ R & \begin{bmatrix} -12 & 18 & 6 \\ 8 & -10 & -4 \\ -6 & 14 & -8 \end{bmatrix} \end{matrix}$$

using $P = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ for $R$'s strategy and using $Q = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$ for $C$'s strategy.

**5.** Here is a payoff matrix for a two-person game:

$$\begin{matrix} & C \\ R & \begin{bmatrix} -15 & 40 & 25 \\ 25 & -10 & -5 \\ 45 & 20 & -15 \end{bmatrix} \end{matrix}$$

Find the expected payoff for each of the pairs of strategies.

**(a)** $P = [1 \quad 0 \quad 0]$ $\quad Q = [0 \quad 1 \quad 0]$

**(b)** $P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ $\quad Q = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$

**(c)** $P = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} & \frac{2}{5} \end{bmatrix}$ $\quad Q = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$

**(d)** $P = [0.3 \quad 0.1 \quad 0.6]$ $Q = [0.2 \quad 0.2 \quad 0.6]$

**The payoff matrices in Exercises 6 through 8 describe mixed-strategy games. Determine the optimal strategies for each game and the resulting value of the game.**

**6.** *(See Example 2)* $\begin{bmatrix} 4 & 9 \\ 10 & 3 \end{bmatrix}$

**7.** $\begin{bmatrix} 15 & 10 \\ -5 & 20 \end{bmatrix}$

**8.** $\begin{bmatrix} 12 & -4 \\ -6 & 14 \end{bmatrix}$

**9.** *(See Example 4)* Reduce the following matrices by deleting rows or columns that are dominated by other rows or columns.

**(a)** $\begin{bmatrix} 6 & -5 & 1 & 9 \\ 2 & 0 & 4 & 7 \\ -3 & -6 & 0 & 3 \end{bmatrix}$ **(b)** $\begin{bmatrix} 5 & 2 & -2 & 8 \\ 3 & 1 & 13 & 22 \\ 1 & 3 & 6 & 11 \end{bmatrix}$

**10.** Reduce the following matrices by deleting rows or columns that are dominated by other rows or columns.

**(a)** $\begin{bmatrix} 6 & 2 & 3 \\ 2 & 0 & 4 \\ 7 & 2 & 1 \\ -3 & -8 & 3 \end{bmatrix}$ **(b)** $\begin{bmatrix} 6 & 2 & -2 & 1 \\ 4 & 1 & 9 & 2 \\ 2 & 4 & 6 & 8 \\ 1 & 3 & 6 & 1 \end{bmatrix}$

**11.** Determine the optimal strategy and the resulting value for this mixed-strategy game:

$$\begin{bmatrix} 6 & 7 & 3 \\ 4 & 5 & 2 \\ 5 & 6 & 8 \end{bmatrix}$$

**12.** Determine the optimal strategy and the resulting value for this mixed-strategy game:

$$\begin{bmatrix} 4 & 6 & 5 \\ 6 & -1 & 7 \\ 8 & 2 & 9 \end{bmatrix}$$

**13.** *(See Example 5)* Determine whether or not the following games are fair.

**(a)** $\begin{bmatrix} 6 & -8 \\ -3 & 4 \end{bmatrix}$ **(b)** $\begin{bmatrix} 5 & 2 \\ 1 & 3 \end{bmatrix}$

**(c)** $\begin{bmatrix} 3 & -9 \\ 2 & 6 \end{bmatrix}$

**14.** Determine whether or not the following games are fair.

**(a)** $\begin{bmatrix} 3 & 2 \\ -1 & 5 \end{bmatrix}$ **(b)** $\begin{bmatrix} 5 & -10 \\ -2 & 4 \end{bmatrix}$

**(c)** $\begin{bmatrix} 7 & 8 \\ 2 & 6 \end{bmatrix}$

## Level 2

**15.** Two television networks, Century and ReMark, are competing for a viewer audience in prime time. Century is considering a new talk show, a game show, and an educational documentary show. ReMark is considering a sports highlights show, a mystery drama, and a variety show. The following payoff matrix shows the estimated gain by ReMark in audience ratings for the pairings of shows.

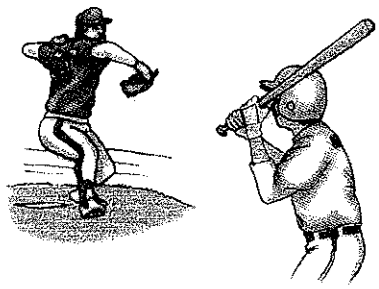
| | | Century | | |
|---|---|---|---|---|
| | | Talk | Game | Educational |
| ReMark | Sports | 10 | −5 | 25 |
| | Mystery | −20 | 15 | 10 |
| | Variety | −10 | −10 | 5 |

The networks are prepared to air their shows in a random manner. Find the optimal strategy of each network and the expected value.

**16.** Baseball has classic one-on-one competition between the pitcher and the batter. Let's analyze the strategy of the pitcher, Perez, and the batter, Blakemore. Perez has two pitches, a fastball and a curve. When Blakemore is set for a fastball and Perez throws a fastball, Blakemore's batting average is 0.450. If, instead, Perez throws a curve, Blakemore's batting average is 0.200. When Blakemore is set for a curve and Perez throws a curve, Blakemore's batting average is 0.350. If Perez throws a fastball instead, Blakemore's batting average is 0.150. Thus, the payoff matrix is

| | | Perez Throws | |
|---|---|---|---|
| | | $F$ | $C$ |
| Blakemore Expects | $F$ | 0.450 | 0.200 |
| | $C$ | 0.150 | 0.350 |

Find the optimal strategy for each player and the expected batting average.



## Level 3

**17.** *(See Example 6)* A patient has symptoms that suggest a disease that requires exploratory surgery to determine if the disease exists. The expected survival times based on whether the disease exists or not ($D$ or $ND$) and on whether or not surgery is performed or not ($S$ or $NS$) are given in the matrix

| | | Nature | |
|---|---|---|---|
| | | $D$ | $ND$ |
| Patient | $S$ | 25 | 30 |
| | $NS$ | 5 | 35 |

The probability that the disease exists is $q_1 = 0.60$.

**(a)** Find the expected survival time with surgery.

**(b)** Find the expected survival time without surgery.

**(c)** For what value of $q_1$ is no surgery the better option?

**18.** A patient faces surgery because of a suspected disease. The expected years of survival are given in the matrix

| | | Nature | |
|---|---|---|---|
| | | $D$ | $ND$ |
| Patient | $S$ | 15 | 35 |
| | $NS$ | 1 | 40 |

The probability that the disease actually exists is $q_1 = 0.35$.

**(a)** Find the expected survival time if surgery is done.

**(b)** Find the expected survival time if surgery is not done.

**(c)** Find the probability $q_1$ for which surgery is the better option.

**19.** A physician tells her patient that symptoms suggest a tumor that requires surgery for further diagnosis and treatment. The following matrix gives expected survival time:

| | | Nature | |
|---|---|---|---|
| | | $D$ | $ND$ |
| Patient | $S$ | 20 | 22 |
| | $NS$ | 3 | 25 |

The physician indicates the probability that the tumor is malignant is considered to be about 0.70, but it can vary from that. Assuming $q_1 = 0.70$,

**(a)** find the expected survival with surgery.

**(b)** find the expected survival without surgery.

**(c)** Because $q_1 = 0.70$ is somewhat questionable, find the range of $q_1$ such that surgery is the better option.

**20.** A patient who has a tumor that is suspected to be malignant has three treatment options: surgery, radiation–chemotherapy, or both. The probability that the tumor is malignant is 0.80. The survival time of the patient under treatment or no treatment options is given for each option.

**Surgery**

| | | Nature | |
|---|---|---|---|
| | | *D* | *ND* |
| Patient | *T* | 20 | 22 |
| | *NT* | 3 | 25 |

**Radiation**

| | | Nature | |
|---|---|---|---|
| | | *D* | *ND* |
| Patient | *T* | 12 | 25 |
| | *NT* | 2 | 30 |

**Both**

| | | Nature | |
|---|---|---|---|
| | | *D* | *ND* |
| Patient | *T* | 18 | 20 |
| | *NT* | 2 | 30 |

Based on expected survival, rank the treatments in order of effectiveness.

Answers

# Chapter G

## Section G.1

**1.**
$$\begin{array}{c} \\ R \\ \\ \end{array}\begin{array}{c} \begin{array}{cc} \quad C & \\ H & T \end{array} \\ \begin{array}{c} H \\ T \end{array}\begin{bmatrix} -1 & -0.5 \\ -0.5 & 2 \end{bmatrix} \end{array}$$

**3.**
$$\begin{array}{c} \\ R \\ \\ \end{array}\begin{array}{c} \begin{array}{cc} \quad C & \\ 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array}\begin{bmatrix} 2 & -3 \\ -3 & 4 \end{bmatrix} \end{array}$$

**5.** **(a)** $R$ receives 10 from $C$. **(b)** 10 **(c)** 10 **(d)** 8 **7.** Strictly determined, the saddle point is $(2, 2)$, value $= -1$, and the solution is row 2 and column 2. **9.** Not strictly determined **11.** Strictly determined, the saddle point is $(1, 2)$, value $= 0$, and the solution is row 1 and column 2. **13.** **(a)** Strictly determined, saddle point $(2, 1)$, value $= 0$, solution row 2, column 1 **(b)** Not strictly determined because the largest row minimum does not equal the smallest column maximum **(c)** Strictly determined, saddle point $(2, 3)$, value $= 3$, solution row 2 and column 3 **(d)** Strictly determined, saddle point $(2, 2)$, value $= 0$, solution row 2, column 2 **15.** ReMark should choose option 1, the sports highlights, and Century should choose option 2, the game show. The value is $-5$. ReMark should expect to lose 5 points in the ratings.

**17.** **(a)** The saddle point is $(1, 1)$, so the farmer should plant milo.

**(c)** Total income over five years:

Milo: $85 + 3(120) + 150 = 595$

Corn: $60 + 3(165) + 235 = 790$

Wheat: $70 + 3(150) + 175 = 695$

In the long term, corn appears to be the best and wheat second.

## Section G.2

**1.** 17.5 **3.** 0 **5.** **(a)** 40 **(b)** 7.5 **(c)** $\frac{34}{3}$ **(d)** 8.4 **7.** Row strategy $= \begin{bmatrix} \frac{5}{6} & \frac{1}{6} \end{bmatrix}$ and column strategy $= \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \end{bmatrix}$, $E = 11.67$ **9.** **(a)** $\begin{bmatrix} -5 \\ 0 \end{bmatrix}$ **(b)** $\begin{bmatrix} 5 & 2 & -2 \\ 3 & 1 & 13 \\ 1 & 3 & 6 \end{bmatrix}$

**11.** $P = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}; Q = \begin{bmatrix} \frac{5}{6} & 0 & \frac{1}{6} \end{bmatrix}; E = \frac{11}{2}$ **13.** **(a)** $E = 0$. This is a fair game. **(b)** $E = \frac{13}{5}$. This is not a fair game. **(c)** $E = \frac{36}{16}$. This is not a fair game. **15.** ReMark should show the sports highlights 70% of the time, the mystery drama 30% of the time, and drop the variety show. Century should air the talk show 40% of the time, the game show 60% of the time, and drop the educational documentary. Using these strategies, ReMark can expect an average gain of 1 rating point. **17.** **(a)** 27 years **(b)** 17 years **(c)** No surgery is the better option when $q_1 < 0.20$.

**19.** **(a)** 20.6 years **(b)** 9.6 years **(c)** Surgery is the better option when the probability of malignancy is greater than 0.15.