Math 152 Exam 2 Review

The following is a collection of questions to review the topics for the second exam. This is not intended to represent an actual exam nor does it have every type of problem seen int he homework.
These questions cover sections 7.3, 7.4, 7.8, 11.1, 11.2, and 11.3.

1. Determine if the series converges. $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n^2+4}$

2. Consider the series $\sum_{n=1}^{\infty} 2ne^{-n^2}$

(a) Show the series converges.

(b) Find an upperbound on the error when using s_4 to approximate the summation.

3. How many terms of the series do we need to add in order to find the sum so that the error is $< \dfrac{1}{20}$?

$$\sum_{n=1}^{\infty} \frac{2}{n^3}$$

4. Give the form of the partial-fraction decomposition for the rational function

$$\frac{x+7}{(x-1)(x^2-1)(x^2+4)} =$$

5. Compute $\int \frac{3x^2 + 4x + 3}{x^2 + 1}\ dx$

6. Compute $\int \frac{1}{\sqrt{x^2+4x}}\, dx$

$\cos^2\theta = 1 - \sin^2\theta$

$\sec^2\theta = 1 + \tan^2\theta$

$\tan^2\theta = \sec^2\theta - 1$

7. Compute $\displaystyle\int \frac{dx}{(x^2+4)^2}$

$\cos^2\theta = 1 - \sin^2\theta$

$\sec^2\theta = 1 + \tan^2\theta$

$\tan^2\theta = \sec^2\theta - 1$

8. Compute $\int \frac{2x^3 + 11x^2 + 18}{x^2(x^2 + 9)}\, dx$

9. Compute $\int_0^4 \frac{1}{(x-4)^4}\ dx$

10. Compute $\int_2^\infty \frac{dx}{x(2x+1)} = \int_2^\infty \frac{1}{x} - \frac{2}{2x+1}\ dx$

11. Use the Comparison Test for Improper Integrals to decide on the convergence/divergence of each of the following improper integrals.

(a) $\displaystyle\int_1^{\infty} \frac{2+e^{-x}}{x}\, dx$

(b) $\displaystyle\int_1^{\infty} \frac{1}{\sqrt{x}+x\sqrt{x}}\, dx$

12. Find a formula for the general term, a_n, of the sequence assuming that the pattern of the first few terms continues. Give the formula so the first term is a_1.

$$\left\{\frac{1}{2}, \frac{-4}{5}, \frac{9}{8}, \frac{-16}{11}, \ldots\right\}$$

13. Determine whether each sequence converges or diverges. If it converges, find the value.

(a) $a_n = \arctan\left(\frac{n^2}{n+5}\right)$

(b) $a_n = \frac{(-1)^n n^2}{2n^2+5}$

14. Assume that the sequence defined below is bounded and is decreasing. Determine if the sequence is convergent or divergent. Give the value the sequence will converge to if it converges.

$$a_1 = 2, \qquad a_{n+1} = \frac{6}{7 - a_n}$$

15. Assume the n-th partial sum of the series $\sum_{n=1}^{\infty} a_n$ is $s_n = \frac{3n^2}{n^2 - 10}$.

(a) Compute a_5.

(b) Determine if $\sum_{n=1}^{\infty} a_n$ converges. If possible, give the sum of the series.

16. Determine if the series converges or diverges. Give the sum if convergent.

(a) $\displaystyle\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right)$

(b) $\displaystyle\sum_{n=5}^{\infty} 10\left(\frac{-2}{5}\right)^{n-1}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{3^{n+2}}{2^{3n+1}}$

(d) $\displaystyle\sum_{k=1}^{\infty}\left[\cos\left(\frac{1}{k+3}\right)-\cos\left(\frac{1}{k+1}\right)\right]$