Spring 2012 Math 151

Week in Review # 9
sections: 5.1, 5.2, 5.3
courtesy: Joe Kahlig

Answer these questions for each of the graphs.

(A) On what intervals is f is increasing? decreasing?
(B) On what intervals is f concave up? concave down?
(C) At what values of x does f have a local maximum or minimum?
(D) At what values of x does f have an inflection point?
(E) Assuming that f is continuous and $f(0) = 0$, sketch a graph of f.

on an interval

$f(x)$ inc $\iff f'(x) > 0$

$f(x)$ dec $\iff f'(x) < 0$

$f(x)$ c.u $\iff f''(x) > 0 \iff f'(x)$ is inc

$f(x)$ c.d $\iff f''(x) < 0 \iff f'(x)$ is dec.

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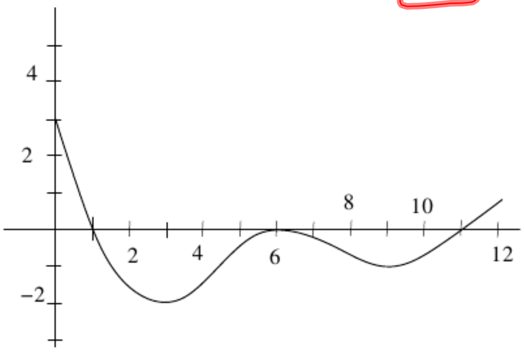
1. The graph of the derivative, $\boxed{f'(x)}$, is shown below.



A) $f(x)$ Inc $(-\infty,1)\cup(11,\infty)$

Dec $(1,6)\cup(6,11)$

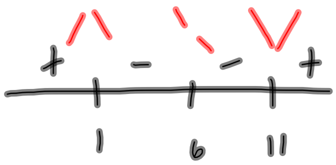
or $(1,11)$

B) $f(x)$

C.U. $(3,6)\cup(9,\infty)$

C.D. $(-\infty,3)\cup(6,9)$

C) f'

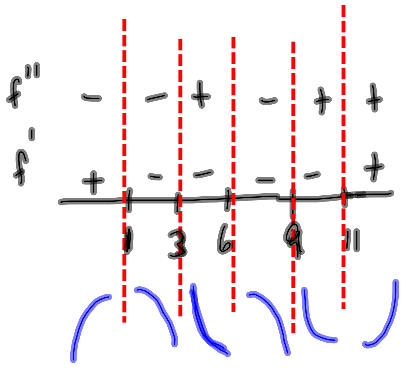


local max @ $x=1$

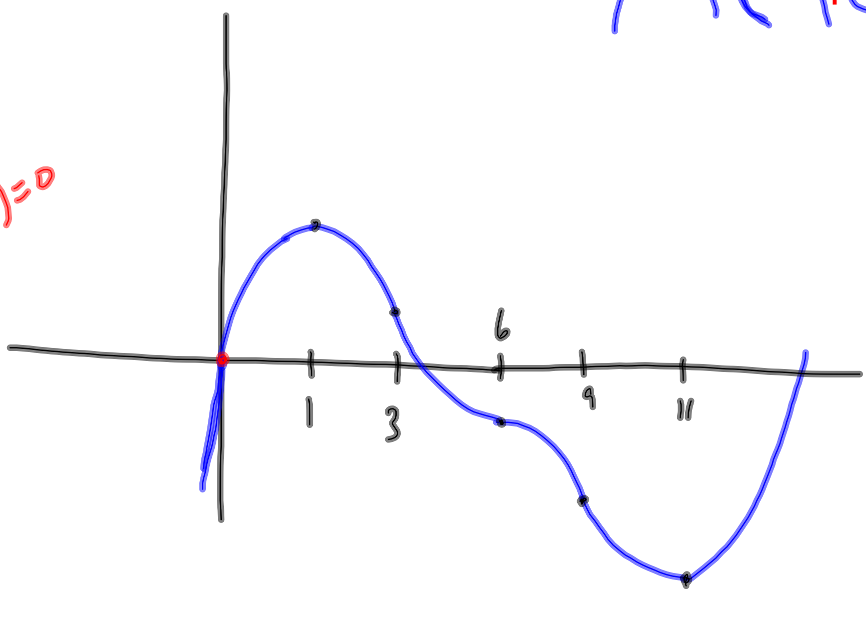
Local min @ $x=11$

critical values are $x=1,6,11$

D) @ $x=3$, $x=6$, $x=9$



$f(0)=0$



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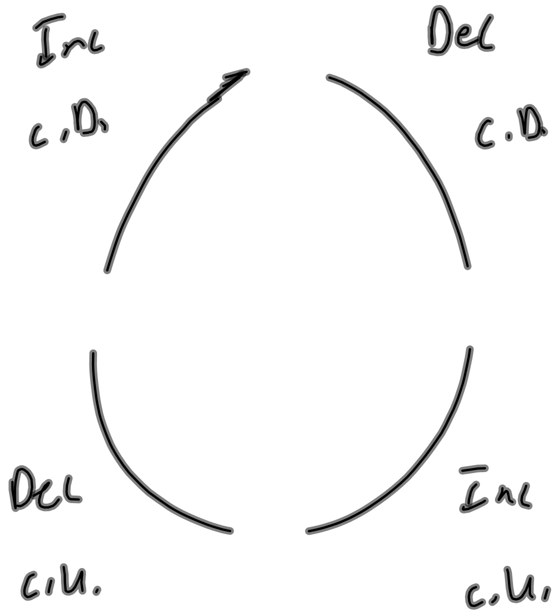
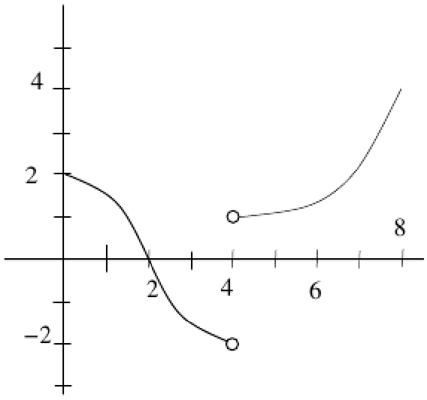

Inc
c.D.
Dec
c.D.
Dec
c.U.
Inc
c.U.

part E says f(x) is cont.

2. The graph of the derivative, $\boxed{f'(x)}$, is shown below.



A) f(x)

Inc $(-\infty, 2) \cup (4, \infty)$

Dec $(2, 4)$

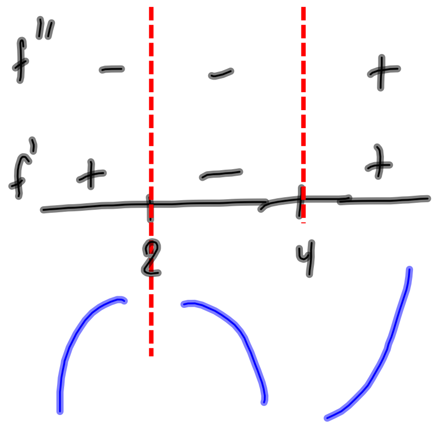
B) f(x)

c.U. $(4, \infty)$

c.D. $(-\infty, 4)$

C) critical values

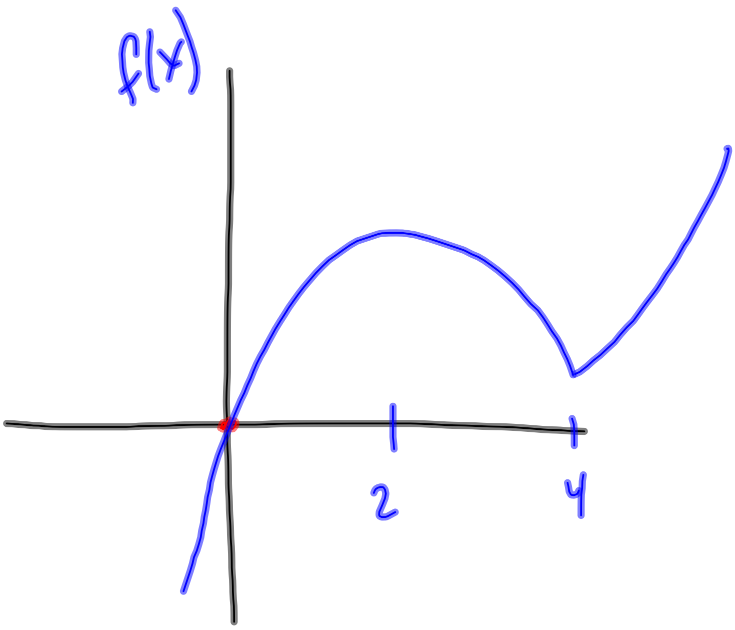
$x = 2$, $x = 4$ (sharp point)



Local max @ $x = 2$

Local min @ $x = 4$

D) Inflection pt @ $x = 4$



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1. For the following functions, find all critical values.

(a) $f(x) = xe^{2x}$

Domain is all Reals.

$f'(x) = 1e^{2x} + x \cdot 2e^{2x}$

$= (1+2x)e^{2x}$

does f' DNE for any x-values? no.

does f' = 0 for any x-values?

$0 = (1+2x)e^{2x}$

$0 = 1+2x$ $\quad$ $\underbrace{0 = e^{2x}}_{\text{not possible}}$

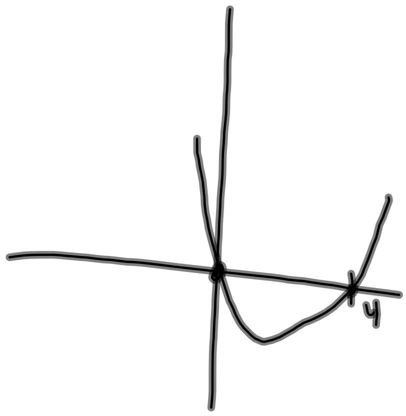
$\boxed{x = -\frac{1}{2}}$

c.v.

(b) $f(x) = |x^2 - 4x| = \begin{cases} x^2-4x, & x \geq 4 \text{ or } x \leq 0 \\ -(x^2-4x), & 0 < x < 4 \end{cases}$

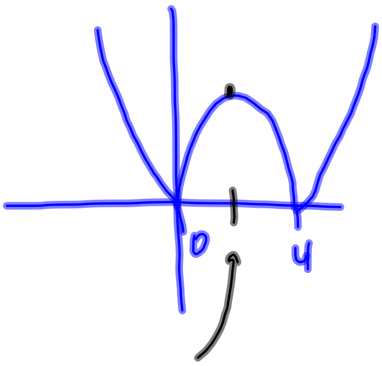
Domain is all Reals

$y = x^2 - 4x$



$f'(x) = \begin{cases} 2x-4, & x>4 \text{ or } x<0 \\ -(2x-4), & 0<x<4 \end{cases}$

$y = |x^2 - 4x|$



$y' = 0$

by the picture y' DNE @ $x=0$ & $x=4$

Thus they are c.v.

c.v. also at $x = 2$

(c) $f(x) = x^{\frac{1}{3}}(8-x)$

Domain all Reals.

$$f' = \frac{1}{3}x^{-\frac{2}{3}}(8-x) + x^{\frac{1}{3}}\cdot(-1)$$

$$= \frac{8-x}{3x^{2/3}} - x^{1/3}$$

$$= \frac{8-x}{3x^{2/3}} - x^{1/3}\cdot\frac{3x^{2/3}}{3x^{2/3}}$$

$$f'(x) = \frac{8-x-3x}{3x^{2/3}} = \frac{8-4x}{3x^{2/3}}$$

$f'(0)$ DNE and $x=0$ is in the domain

thus $x=0$ is a C.V.

$$0 = \frac{8-4x}{3x^{2/3}}$$

$0 = 8-4x$

$4x = 8$

$x = 2$

$x = 2$ C.V.

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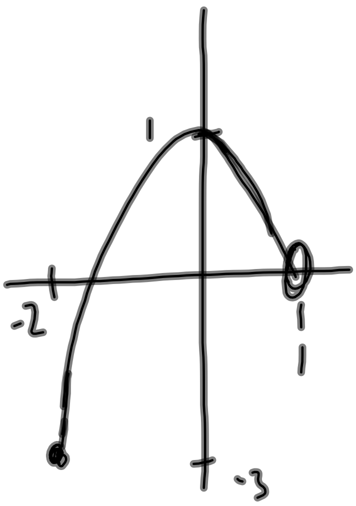
2. Find the absolute and local extrema for these functions by graphing.

(a) $f(x) = 1 - x^2$, $\underbrace{-2 \le x < 1}$



abs max = 1 (y-values)

abs min = -3 (y-values)

local max = 1 (happens at x=0)

local min none.

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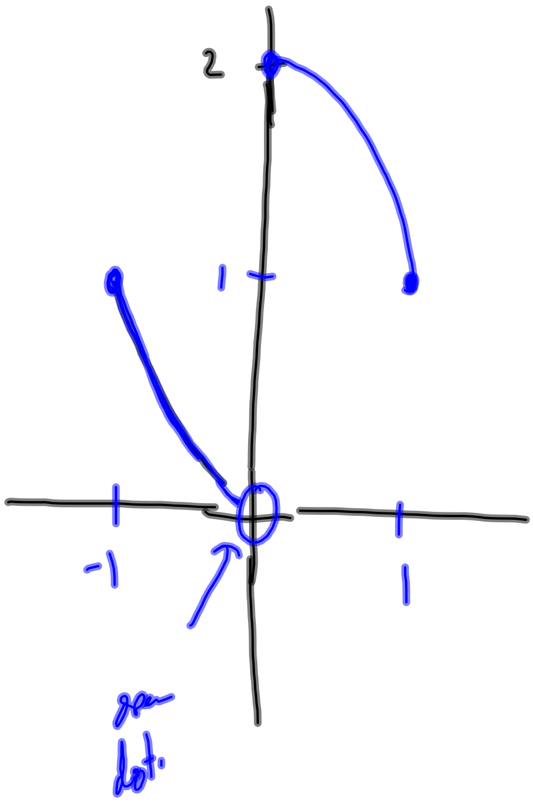
(b) $f(x) = \begin{cases} x^2, & \text{if } -1 \leq x < 0 \\ 2 - x^2, & \text{if } 0 \leq x \leq 1 \end{cases}$



Abs max = 2

Abs min ONE

C.V. @ x=0

Local max @ x=0

3. Find the absolute maximum and absolute minimum of the given function on the given interval

(a) $f(x) = x^3 - 2x^2 + x - 5$ on $[-1, 3]$

$f(x)$ cont ✓

Interval closed ✓

includes the end points

$f' = 3x^2 - 4x + 1$

$0 = 3x^2 - 4x + 1$

$0 = (3x - 1)(x - 1)$

C.V. $x = \frac{1}{3}$ $x = 1$

$f(-1) = -9$

$f\left(\frac{1}{3}\right) = -4.852$

$f(1) = -5$

$f(3) = 7$

abs max = 7

abs min = -9

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(b) $f(x) = x^{\frac{5}{3}} + 5x^{\frac{2}{3}}$ on $[-1, 4]$

$$f' = \frac{5}{3} x^{2/3} + 5 \cdot \frac{2}{3} x^{-1/3}$$

$$= \frac{5}{3} x^{2/3} + \frac{5}{3} \cdot 2x^{-1/3}$$

$$f' = \frac{5}{3} x^{-\frac{1}{3}} (x + 2)$$

$$f' = \frac{5}{3} \frac{x+2}{\sqrt[3]{x}}$$

$x = 0$ is a C.V.

$x = -2$ is a C.N.

$$0 = \frac{5}{3} \frac{x+2}{\sqrt[3]{x}}$$

$$0 = 5(x+2)$$

$$0 = x + 2$$

C.V. $x = -2$

$f(x) = x^{5/3} + 5x^{2/3}$

Interval $[-1, 4]$

$f(-1) = 4$

$f(0) = 0$

$f(4) = 22.679$

abs max $= 22.679$

abs min $= 0$

(c) $f(x) = \frac{1}{(x-1)^2}$, on $[0,3]$

not cont at $x=1$

V.A. @ $x=1$

$\lim_{x \to 1^+} f(x) = +\infty$

$\lim_{x \to 1^-} f(x) = +\infty$ ⟶ no abs max.

$f(x) = (x-1)^{-2}$

$f' = -2(x-1)^{-3} \cdot 1 = \frac{-2}{(x-1)^3}$

$f' \neq 0$. since $\frac{-2}{(x-1)^3} = 0$ has no solution.

Thus no. C.V.

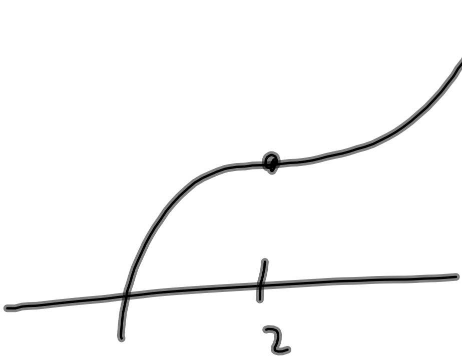
Test endpoints

$f(0) = 1$

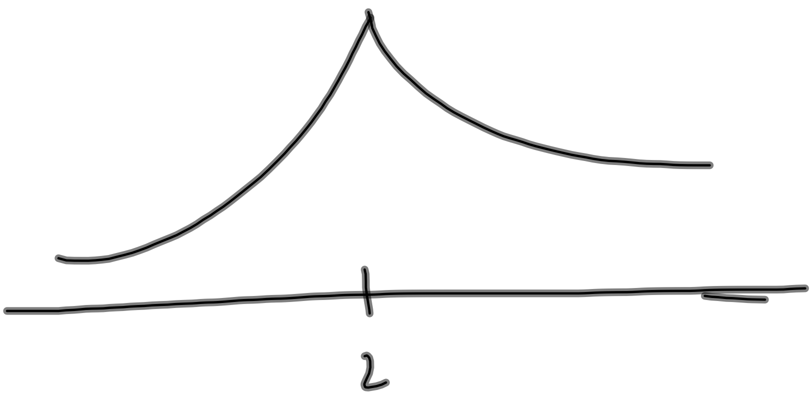
$f(3) = \frac{1}{4}$ ⟶ abs min $= \frac{1}{4}$

4. Sketch a graph of a function f satisfying the following conditions.

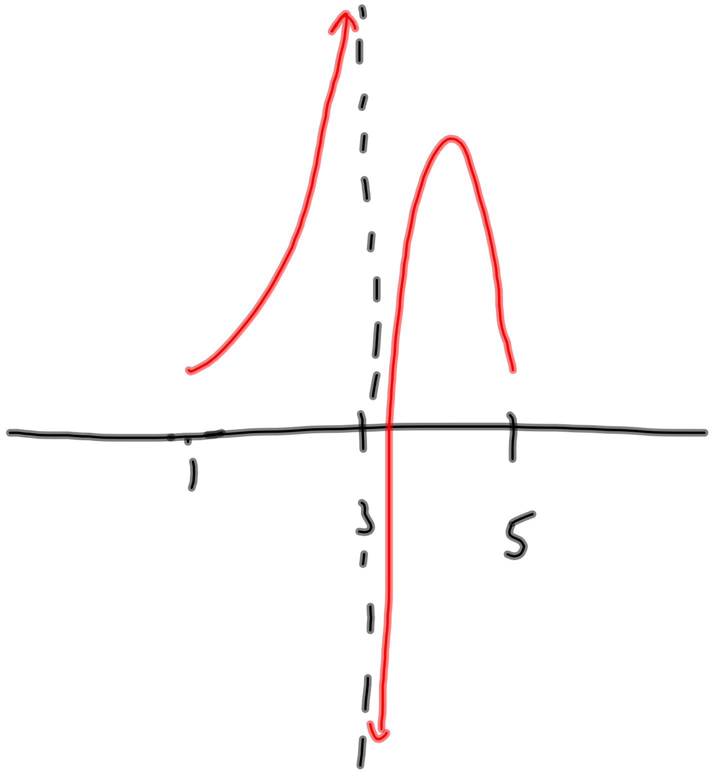
(a) $x = 2$ is a critical number, but f has no local extrema.



(b) f is continuous with a local maximum at $x = 2$, but f is not differentiable at $x = 2$.



(c) f is defined on the interval $[1, 5]$ but does not have an absolute maximum.



5. Find the value of c in the interval $[1, 4]$ that satisfies the conclusion of the Mean Value Theorem
for $f(x) = x^3 + 5$

$f' = 3x^2$

$f'(c) = \dfrac{f(b) - f(a)}{b - a}$

with $a < c < b$

$3c^2 = \dfrac{f(4) - f(1)}{4 - 1}$

$3c^2 = \dfrac{69 - 6}{3}$

$3c^2 = \dfrac{63}{3}$

$3c^2 = 21$

$c^2 = 7$

$c = \pm\sqrt{7}$

on $[1, 4]$

$c = \sqrt{7}$

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6. Find the intervals where the function is increasing or decreasing and identify all local extrema.

(a) $f(x) = xe^{x^2-3x}$

Domain all Real #s.

$$f' = e^{x^2-3x} + x\cdot(2x-3)e^{x^2-3x}$$

$$= \left(1 + x(2x-3)\right)e^{x^2-3x}$$

$$f' = (1 + 2x^2 - 3x)\,e^{x^2-3x}$$

f': $+$ on $(-\infty, \tfrac{1}{2})$, $-$ on $(\tfrac{1}{2}, 1)$, $+$ on $(1, \infty)$; test points 0, $.6$, 10; marks at $\tfrac{1}{2}$ and 1

set $f' = 0$

$$0 = (2x^2 - 3x + 1) \qquad \underbrace{0 = e^{x^2-3x}}_{\text{doesn't happen}}$$

$$0 = (2x-1)(x-1)$$

C.V. $x = \frac{1}{2}$ $\quad x = 1$

Inc $(-\infty, \frac{1}{2}) \cup (1, \infty)$

Dec $(\frac{1}{2}, 1)$

Rel max @ $x = \frac{1}{2}$

Rel min @ $x = 1$

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(b) $f(x) = \dfrac{x}{(x-1)^2}$

Domain is all Reals except $x \neq 1$

$$f' = \frac{(x-1)^2(1) - x \cdot 2(x-1)' \cdot 1}{(x-1)^4}$$

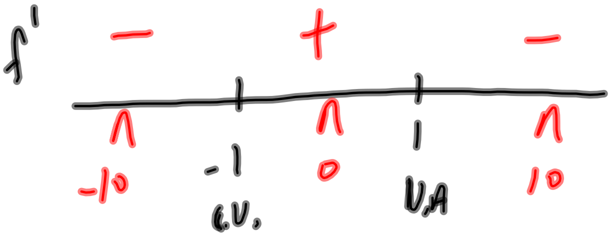
$$= \frac{(x-1)\left[(x-1) - 2x\right]}{(x-1)^4} = \frac{x-1-2x}{(x-1)^3} = \frac{-x-1}{(x-1)^3} = f'$$

$f' = 0$

$$\frac{-x-1}{(x-1)^3} = 0$$

$$-x-1 = 0$$

C.V. $-1 = x$



Inc $(-1, 1)$

Dec $(-\infty, -1) \cup (1, \infty)$

Local min @ $x = -1$

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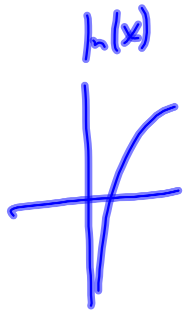
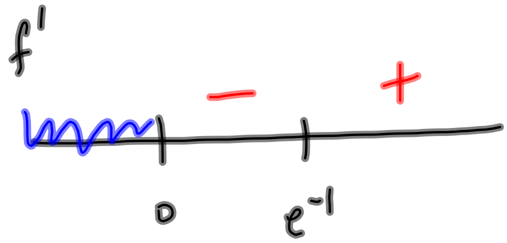
(c) $f(x) = x \ln(x)$

Domain is $x > 0$



$f' = 1 \ln(x) + x \cdot \frac{1}{x}$

$f' = \ln(x) + 1$



$0 = \ln(x) + 1$

$-1 = \ln(x)$

C.V. $e^{-1} = x$

Inc (e^{-1}, ∞)

Dec $(0, e^{-1})$

Local min @ $x = e^{-1}$

(d) $f(x) = x \sin x + \cos x$ on $[0, 2\pi]$

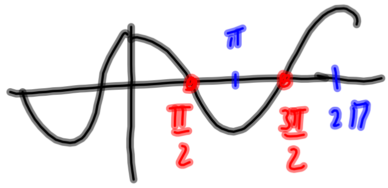
$f'(x) = \sin(x) + x\cos(x) - \sin(x)$

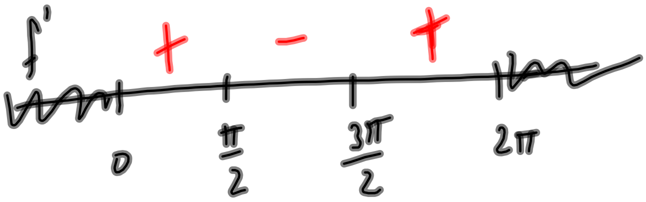
$f'(x) = x\cos(x)$

$0 = x\cos(x)$

$x = 0 \qquad \cos(x) = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$





Inc $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)$

Dec $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

Local max @ $x = \frac{\pi}{2}$

Local min @ $x = \frac{3\pi}{2}$

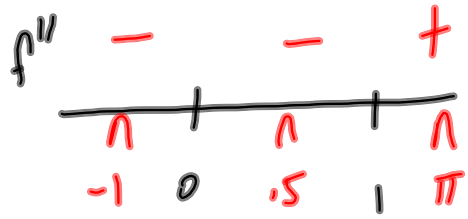
7. Determine the intervals where the given function, $f(x)$ is concave up or concave down and identify all inflection points:

(a) $f(x) = 5x^7 - 7x^6 + 10$

$f' = 35x^6 - 42x^5$

$f'' = 210x^5 - 210x^4$

$f'' = 210x^4(x-1)$



$0 = 210x^4(x-1)$

$x=0 \quad x=1$

C.D $(-\infty,0)\cup(0,1)$

C.U $(1,\infty)$

Inflection pt @ $x=1$

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(b) $f(x) = x\ln(x-2)$

Domain is $x > 2$

$$f' = 1 \cdot \ln(x-2) + x \cdot \frac{1}{x-2}$$

$$= \ln(x-2) + \frac{x}{x-2}$$

$$f'' = \frac{1}{x-2} + \frac{(x-2)(1) - x(1)}{(x-2)^2}$$

$$= \frac{1}{x-2} + \frac{x-2-x}{(x-2)^2} = \frac{1}{x-2} + \frac{-2}{(x-2)^2}$$

$$f'' = \frac{x-2-2}{(x-2)^2} = \frac{x-4}{(x-2)^2}$$

$$0 = \frac{x-4}{(x-2)^2}$$

$0 = x-4$

$x = 4$

f'': $-$ (at 3), $+$ (at 5)

2 V.A, 3, 4, 5

C.U. $(4, \infty)$

C.D $(2, 4)$

Inflection pt @ $x = 4$

8. Given $f(3) = 8$, $f'(3) = 0$, $f''(3) = 6$, $f(7) = 1$, $f'(7) = 0$, and $f''(7) = -5$, identify any local extrema of f.

$x=3$ & $x=7$ are C.V.

3 & 7 in domain of $f(x)$

$f''(3) = 6$ C.U. @ $x=3$ $\cup$

Local min @ $x=3$

$f''(7) = -5$ C.D @ $x=7$ $\cap$

Local max @ $x=7$

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9. Find the values of A and B so that the function $f(x) = Ax^3 - 36x^2 + Bx + 7$ will have an inflection point at $x = 3$ and will have a rate of change of -36 at $x = 2$.

$f''(3) = 0$

$f'(2) = -36$

$f' = 3Ax^2 - 72x + B = 12x^2 - 72x + B$

$f'' = 6Ax - 72$

$f''(3) = 0$

$0 = 6A(3) - 72$

$0 = 18A - 72$

$72 = 18A$

$A = 4$

$f'(2) = -36$

$-36 = 12(2)^2 - 72(2) + B$

$-36 = 48 - 144 + B$

$-36 = -96 + B$

$60 = B$

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