Spring 2012 Math 151

Week in Review # 7
sections: 4.2, 4.3, 4.4
courtesy: Joe Kahlig

if $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$

if $f(x_1) = f(x_2)$ then $x_1 = x_2$

1. Determin if the function is one-to-one.
$y = \sqrt[3]{x^2+1}$

not one-to-one.

$X = 1 \qquad y = \sqrt[3]{2}$

$X = -1 \qquad y = \sqrt[3]{2}$

$f(0) = \frac{1}{5}$ if $f(a) = f(b)$ then $a = b$

2. Show that the function is one-to-one and find the inverse.

(a) $f(x) = \dfrac{1-4x}{3x+5}$

Show 1-1 (one-to-one)

$f(a) = f(b)$

$$\frac{1-4a}{3a+5} = \frac{1-4b}{3b+5}$$

$$(1-4a)(3b+5) = (1-4b)(3a+5)$$

$$3b + \underbrace{5 - 12ab}_{} - 20a = 3a + \underbrace{5 - 12ab}_{} - 20b$$

$$3b - 20a = 3a - 20b$$

$$23b = 23a$$

$$b = a \quad \checkmark$$

Thus $f(x)$ is 1-1

find the inverse

$$f(x) = y = \frac{1-4x}{3x+5}$$

step 1

$$x = \frac{1-4y}{3y+5}$$

step 2: solve for y.

$$x(3y+5) = 1-4y$$

$$3xy + 5x = 1 - 4y$$

$$3xy + 4y = 1 - 5x$$

$$y(3x+4) = 1-5x$$

$$f^{-1}(x) = y = \frac{1-5x}{3x+4}$$

$$f(0) = \frac{1}{5}$$

$$f^{-1}\left(\frac{1}{5}\right) = 0$$

$$f^{-1}\left(\frac{1}{5}\right) = \frac{1-5\left(\frac{1}{5}\right)}{3\left(\frac{1}{5}\right)+4} = 0$$

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2. Show that the function is one-to-one and find the inverse.

(b) $f(x) = \sqrt[3]{x^3 + 7}$

$f(a) = f(b)$

$\sqrt[3]{a^3+7} = \sqrt[3]{b^3+7}$

$a^3+7 = b^3+7$

$a^3 = b^3$

$a = b$ ✓

find the inverse

$x = \sqrt[3]{y^3+7}$

$x^3 = y^3+7$

$x^3 - 7 = y^3$

$y = \sqrt[3]{x^3-7}$

$\boxed{f^{-1}(x) = \sqrt[3]{x^3-7}}$

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3. Assume that the function f is a one-to-one function with inverse g. Compute a formula for $g'(x)$ based on f.

$g = f^{-1}$

$g(x) = \_\_$

$f(\_) = x$

$$f(g(x)) = x$$

$$f'(g(x)) \cdot g'(x) = 1$$

$$\boxed{g'(x) = \frac{1}{f'(g(x))}}$$

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4. If g is the inverse of the function f, compute $g'(2)$.

$f(x) = x^5 - x^3 + 2x$

$f'(x) = 5x^4 - 3x^2 + 2$

$g(2) = \underline{1}$

$f(1) = 2$

$$g'(2) = \frac{1}{f'(g(2))}$$

$$= \frac{1}{f'(1)} = \frac{1}{5-3+2} = \frac{1}{4}$$

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5. If g is the inverse of the function f, compute $g'(6)$.

$f(x) = 5 + xe^{(x^2-2x+1)}$

$$g'(6) = \frac{1}{f'(g(6))}$$

$$= \frac{1}{f'(1)}$$

$$= \frac{1}{1} = 1$$

$$g(6) = \underline{1}$$

$$f(\underline{1}) = 6$$

$$f'(x) = e^{(x^2-2x+1)} + x\cdot(2x-2)e^{x^2-2x+1}$$

$$f'(1) = e^0 + 1\cdot(2(1)-2)\,e^0$$

$$= 1 + 0 = 1$$

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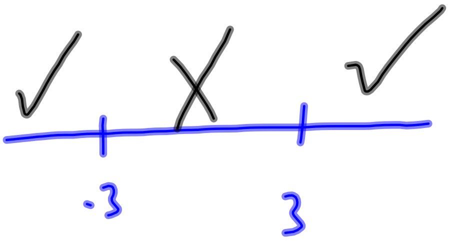
6. Evaluate: $4^{2\log_4 9} = 4^{\log_4 9^2} = 4^{\log_4 81} = 81$

7. Find the domain of this function.

$y = \ln(x^2 - 9)$

$x^2 - 9 = 0$

$x = \pm 3$



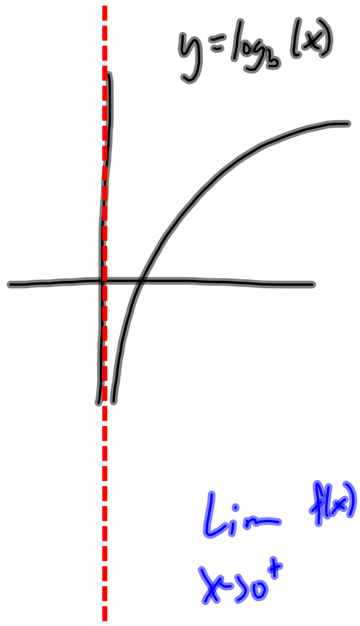
Domain is $(-\infty, -3) \cup (3, \infty)$

VA $x = -3$ & $x = 3$

$y = \log_b(x)$

Domain is $x > 0$

$y = \log_b(x)$ with $b > 1$



$\lim_{x \to 0^+} f(x) = -\infty$

VA $x = 0$

$\log_a 2 = .38$

$a^{.38} = 2$

$\log ab = \log a + \log b$

$\log\left(\frac{a}{b}\right) = \log a - \log b$

$\log a^n = n \log a$

8. Use the fact that $\log_a 2 = 0.38$, $\log_a 3 = 0.63$, and $\log_a 5 = 0.88$ to compute these logarithms.

(a) $\log_a 20 = \log_a 4\cdot 5 = \log_a 4 + \log_a 5$

$= \log_a 2^2 + \log_a 5$

$= 2\log_a 2 + \log_a 5$

$= 2(.38) + .88 = 1.64$

(b) $\log_a\left(\frac{81}{a^3}\right) = \log_a 81 - \log_a a^3$

$= \log_a 3^4 - \log_a a^3$

$= 4\log_a 3 - 3$

$= 4(.63) - 3 = -.48$

9. Solve for x.

(a) $4e^{(3x+2)} = 12$

$\ln(x) = \log_e x$

$\log(x) = \log_{10} x$

$e^{3x+2} = 3$

$\ln\left(e^{3x+2}\right) = \ln(3)$

$3x+2 = \ln(3)$

$3x = \ln(3) - 2$

$\boxed{x = \frac{\ln(3)-2}{3}}$

$\log e^{3x+2} = \log 3$

$(3x+2)\log e = \log 3$

$3x+2 = \frac{\log 3}{\log e}$

$3x = -2 + \frac{\log 3}{\log e}$

$\boxed{x = \frac{1}{3}\left[-2 + \frac{\log 3}{\log e}\right]}$

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(b) $5(7)^{4x} = 3$

$7^{4x} = \frac{3}{5}$

$7^{4x} = .6$

$\log_7 7^{4x} = \log_7(.6)$

$4x = \log_7(.6)$

$x = \frac{1}{4} \log_7(.6)$

$\ln 7^{4x} = \ln(.6)$

$4x \ln(7) = \ln(.6)$

$x = \frac{1}{4} \cdot \frac{\ln(.6)}{\ln(7)}$

(c) $\log(x-2)+\log(x+4)=\log 7$

$$\log[(x-2)(x+4)] = \log 7$$

$$(x-2)(x+4) = 7$$

$$x^2+2x-8 = 7$$

$$x^2+2x-15=0$$

$$(x+5)(x-3)=0$$

$x=-5$ ✗

$\boxed{x=3}$ ✓

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(d) $\log_x(6x-5)=2$

$$x^{\log_x(6x-5)} = x^2$$

$$x^2 = 6x-5$$

$$x^2-6x+5=0$$

$$(x-5)(x-1)$$

$$\boxed{x=5} \checkmark \qquad \cancel{x=1}$$

Base of a logarithm can not be 1 (or a neg. #)

(e) $\log_{27}(\boxed{4\log_2(5x-4)-17}) = \frac{1}{3}$

$$27^{1/3} = 4\log_2(5x-4) - 17$$

$$3 = 4\log_2(5x-4) - 17$$

$$20 = 4\log_2(5x-4)$$

$$5 = \log_2(5x-4)$$

$$2^5 = 5x-4$$

$$32 = 5x-4$$

$$36 = 5x$$

$$\boxed{x = \frac{36}{5}}$$

$$y = e^{f(x)} \qquad y' = f'(x)\, e^{f(x)}$$

$$y = a^{f(x)} \qquad y' = f'(x)\, a^{f(x)} \cdot \ln(a)$$

$$y = \ln(f(x)) \qquad y' = \frac{f'(x)}{f(x)}$$

$$y = \log_a(f(x)) \qquad y' = \frac{f'(x)}{f(x)\ln(a)}$$

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For problem 10-16, find the derivatives of these functions.

10. $y = \log_5(7 - 4x) + 3^{sec(x)}$

$$y' = \frac{-4}{(7-4x)\ln(5)} + \sec(x)\tan(x) \cdot 3^{\sec(x)} \cdot \ln(3)$$

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11. $y = [\ln(x^4 + 5x)]^{\frac{4}{3}}$

$$y' = \frac{4}{3}\left(\ln\left(x^4+5x\right)\right)^{1/3} \cdot \frac{4x^3+5}{x^4+5x}$$

12. $y = \ln(\boxed{\ln(3x+1)})$

$$y' = \frac{1}{\ln(3x+1)} \cdot \frac{3}{(3x+1)} = \frac{3}{(3x+1)\ln(3x+1)}$$

13. $y = 7^{x^2} \log(x^4 + 1)$

$$y' = \underbrace{2x \cdot 7^{x^2} \cdot \ln(7)} \cdot \log(x^4+1) + 7^{x^2} \cdot \frac{4x^3}{(x^4+1)\ln(10)}$$

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14. $y = \ln\sqrt{\dfrac{x^2+5}{5x-8}}$ $= \ln\left(\dfrac{x^2+5}{5x-8}\right)^{1/2}$

$$= \frac{1}{2}\ln\left(\frac{x^2+5}{5x-8}\right) = \frac{1}{2}\left[\ln(x^2+5) - \ln(5x-8)\right]$$

$$y = \frac{1}{2}\ln(x^2+5) - \frac{1}{2}\ln(5x-8)$$

$$y' = \frac{1}{2}\,\frac{2x}{x^2+5} - \frac{1}{2}\cdot\frac{5}{5x-8}$$

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15. $y = (x^2+3)^{\cos(2x)}$

$$\ln y = \ln\left((x^2+3)^{\cos(2x)}\right)$$

$$\ln y = \cos(2x)\ln(x^2+3)$$

$$\frac{y'}{y} = -2\sin(2x)\ln(x^2+3) + \cos(2x)\cdot\frac{2x}{x^2+3}$$

$$y' = y\left[-2\sin(2x)\ln(x^2+3) + \cos(2x)\cdot\frac{2x}{x^2+3}\right]$$

$$y' = (x^2+3)^{\cos(2x)}\left[-2\sin(2x)\ln(x^2+3) + \frac{2x\cos(2x)}{x^2+3}\right]$$

16. $y = \dfrac{(2x^6+4)^5}{(7x^2+5)^3}$

$$\ln(y) = \ln\left(\frac{(2x^6+4)^5}{(7x^2+5)^3}\right)$$

$$\ln(y) = 5\ln(2x^6+4) - 3\ln(7x^2+5)$$

$$\frac{y'}{y} = 5 \cdot \frac{12x^5}{2x^6+4} - 3 \cdot \frac{14x}{7x^2+5}$$

$$y' = y \cdot \left[\frac{60x^5}{3x^6+4} - \frac{42x}{7x^2+5}\right]$$

$$y' = \frac{(2x^6+4)^5}{(7x^2+5)^3}\left[\frac{60x^5}{2x^6+4} - \frac{42x}{7x^2+5}\right]$$