Spring 2012 Math 151

Week in Review # 3

sections: 2.3, 2.5, 2.6

courtesy: Joe Kahlig

Section 2.3

Compute the exact values of these limits. If the limit doesn't exist, support your answer by evaluating left and right hand limits.

1. $\lim_{x\to -1} \dfrac{x^2-x-2}{x^2+9x+8} =$

2. $\lim_{x\to 2^+} \dfrac{x+3}{x^2-4x+4} =$

3. Use the function $f(x)$ to evaluate these limits. $f(x) = \begin{cases} \sqrt{x^2+16}, & \text{if } x \le 3 \\ x^3-10, & \text{if } x > 3 \end{cases}$

   (a) $\lim_{x\to 2} f(x)$

   (b) $\lim_{x\to 3^+} f(x)$

   (c) $\lim_{x\to 3} f(x)$

4. $\lim_{x\to 2} \dfrac{|3x-6|}{x-2} =$

5. $\lim_{x\to 0} \dfrac{(4+x)^{-1}-4^{-1}}{x} =$

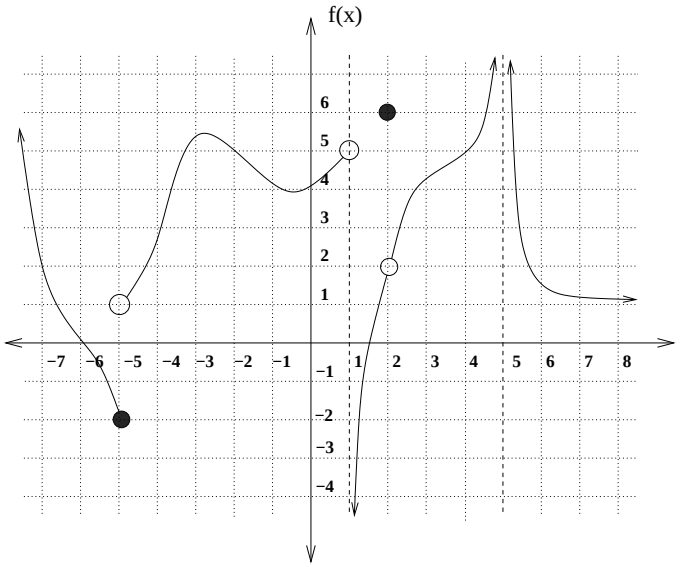
6. $\lim_{x\to 3} \dfrac{x-\sqrt{4x-3}}{x^2-9}$

7. $\lim_{x\to 0^-} \left(\dfrac{1}{x} - \dfrac{1}{|x|}\right)$

8. If $3x \le f(x) \le x^3+2$ for $0 \le x \le 2$, evaluate $\lim_{x\to 1} f(x)$.

Section 2.5

9. Explain, using the definition of continuity, why the function $f(x)$ is continuous or is not continuous at $x = -5$, $x = 2$, and $x = 4$.



10. Find the value(s) of x where the function $f(x)$ is discontinuous. If the discontinuity, $x = a$, is removable, find a function g that agrees with f for all values of x and is continuous at $x = a$.

    (a) $f(x) = \dfrac{x+5}{x^2+x-20}$.

    (b) $f(x) = \dfrac{x^2(x-4)(x+5)}{x(x-4)^2(x+10)}$

11. Where is the function $f(x)$ not continuous. Support your answer.

$$f(x) = \begin{cases} 3x-4 & \text{if } x > 2 \\ x & \text{if } -1 < x \leq 2 \\ 2-x & \text{if } x \leq -1 \end{cases}$$

12. If $g(x) = x^5 - 3x^4 + 2x^2 - 2$, use the Intermediate Value Theorem to find an interval which contains a solution to the equation $g(x) = 1$

13. Use the Intermediate Value Theorem to find two consecutive integers a and $a+1$ such that the interval $[a, a+1]$ contains a solution to the equation $x^3 - 5x - 5 = 0$

14. Find the values of A and B that will make the function $f(x)$ continuous for all real numbers.

$$f(x) = \begin{cases} x^2+5 & \text{if } x < -1 \\ Ax^2+Bx+2 & \text{if } -1 \leq x \leq 2 \\ -9x & \text{if } x > 2 \end{cases}$$

Section 2.6

15. Compute these limits.

(a) $\lim_{x\to\infty} \dfrac{x^2+4x-5}{4x^2-x-3}$

(b) $\lim_{x\to-\infty} \dfrac{2x^3-4x^2+1}{2x^4-3x^2+5}$

(c) $\lim_{x\to\infty} \dfrac{3x^3-4x^6}{-2x^3+x^2+3}$

(d) $\lim_{x\to\infty} \dfrac{\sqrt{3+2x+7x^2}}{3x+5}$

(e) $\lim_{x\to-\infty} \dfrac{\sqrt{3x^2+2}}{5x-4}$

(f) $\lim_{x\to\infty} \sqrt{49x^2-5x+1}-7x$

16. Find all horizontal and vertical asymptotes for $f(x) = \dfrac{x^2+4x-5}{4x^2-x-3}$