Spring 2012 Math 151

Week in Review # 11
sections: 6.1, 6.2, 6.3, and 6.4
courtesy: Joe Kahlig

Section 6.1

If c is any constant (that does not depend on i) and m and n are positive integer with $m < n$, then

$$\sum_{i=m}^{n} ca_i = c\sum_{i=m}^{n} a_i \qquad \sum_{i=m}^{n} (a_i \pm b_i) = \sum_{i=m}^{n} a_i \pm \sum_{i=m}^{n} b_i$$

$$\sum_{i=1}^{n} c = cn \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

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1. Write the sum in sigma notation.
   $\frac{5}{2} + \frac{6}{3} + \frac{7}{4} + \frac{8}{5} + ... + \frac{15}{12}$

2. Find the value of the sum.

   (a) $\displaystyle\sum_{i=1}^{60}(i^2+2)$

   (b) $\displaystyle\sum_{i=10}^{50} i$

   (c) $\displaystyle\sum_{i=3}^{100}\left(\frac{1}{2i-1} - \frac{1}{2i+1}\right)$

3. Suppose it is known that $\displaystyle\sum_{n=1}^{50} k(n) = 25$. Find $\displaystyle\sum_{n=1}^{50}(6k(n)+n)$.

Section 6.2

4. Use the function $f(x) = x^2 + 4$ and the partition $P = \{0, 2, 6, 12, 15\}$ where x_i^* is the right endpoint to answer the following.

   (a) Find the norm of the partition.

   (b) Sketch the function f and the approximation rectangles.

   (c) Find the sum of the approximating rectangles.

Section 6.3

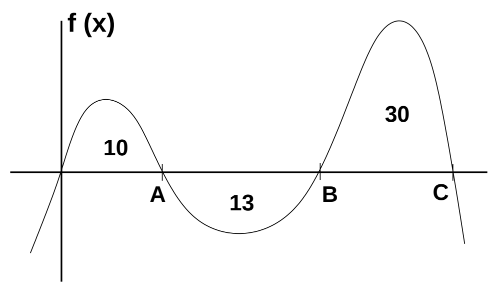
5. Use the midpoint rule with $n = 4$ to approximate $\int_1^9 \ln(x)\ dx$

6. Calculate these definite integrals by referring to the graph of $f(x)$. The areas of the regions are indicated on the graph.

   (a) $\int_0^A f(x)\ dx$

   (b) $\int_A^C f(x)\ dx$

   (c) $\int_C^B f(x)\ dx$



7. Use areas to evaluate this definite integrals.

$$\int_0^4 |5x - 8|\ dx$$

8. If $\displaystyle\int_A^B f(x)\ dx = 12$, $\displaystyle\int_B^A h(x)\ dx = 15$ and

$\displaystyle\int_A^B [2f(x) - 3g(x) + 5h(x)]\ dx = 150$, find $\displaystyle\int_A^B g(x)\ dx$.

9. Express the the definite integral as a limit of a Riemann sum.

$\displaystyle\int_1^8 \sqrt{x+5}\ dx$

10. Express the following limits as definite integrals.

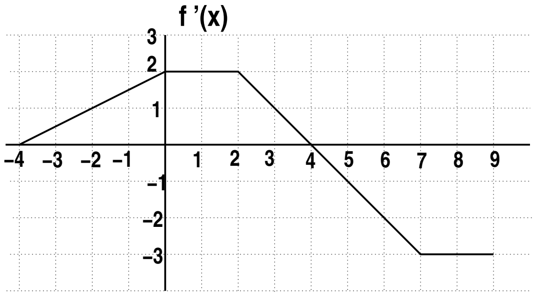
(a) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left[4\left(1+\frac{2i}{n}\right)^3 + 1\right]$

(b) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{8}{n}\left[\frac{1}{8+\left(3+\frac{8i}{n}\right)^2}\right]$

11. Find values for$M$ and J so that $J \le \displaystyle\int_0^2 \sqrt{x^3+1}\ dx \le M$ Be sure to show justification to the choice of M and J that are used.

Section 6.4

12. Use the graph of $f'(x)$ and the fact that $f(0) = 5$ to compute $f(3)$ and $f(8)$



13. Find the derivatives of these functions.

(a) $g(x) = \int_{x^2}^{5} \frac{t}{t^3+1}\, dt$

(b) $f(x) = \int_{\tan(4x)}^{3x+4} e^{t^2}\, dt$

14. Compute these integrals.

(a) $\int_{1}^{4} (3x+1)(x-1)\, dx$

(b) $\int_{2}^{5} \frac{8x+3}{x^2}\, dx$

(c) $\int_0^5 \sin(x) + 3e^x \, dx$

(d) $\int_0^1 \frac{6}{1+x^2} \, dx$

15. The velocity function (in meters per second) for a particle moving along a line is given by $v(t) = t^2 - 6t + 8$ for $0 \leq t \leq 6$. Assume that the particle is moving to the right when the velocity is positive.

(a) Find the displacement by the particle during this time interval.

(b) Find the distance traveled by the particle during this time interval.