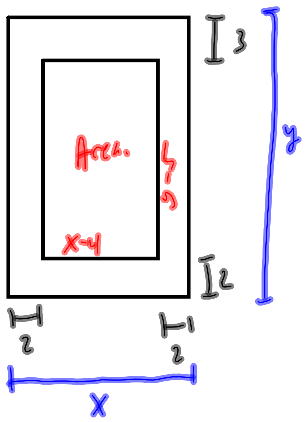
Spring 2012 Math 151

Week in Review # 10
sections: 5.5, 5.7, and 6.1
courtesy: Joe Kahlig

1. A poster is to have an area of 240 in^2 with 2-inch margins at the bottom and the sides and a 3-inch margin at the top. What dimensions will give the largest printed area?



$240 = xy$

$\frac{240}{x} = y$

$5\overline{)240}$ 48

$A = (x-4)(y-5)$

$A = (x-4)\left(\frac{240}{x} - 5\right)$

$= 240 - 5x - \frac{960}{x} + 20$

$A = 260 - 5x - \underbrace{\frac{960}{x}}_{960x^{-1}}$

Domain: $[4, 48]$

$A' = -5 + \frac{960}{x^2}$

find c.v. $A' = 0$

$0 = -5 + \frac{960}{x^2}$

$5 = \frac{960}{x^2}$

$5x^2 = 960$

$x^2 = 192$

$x = \pm\sqrt{192} = \pm 8\sqrt{3}$

c.v. $x = 8\sqrt{3}$

$A'' = \frac{-1920}{x^3}$

$A''(8\sqrt{3}) < 0$

by 2nd deriv. test

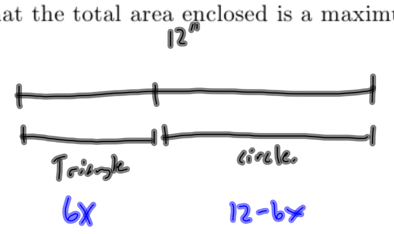
The c.v. is a max

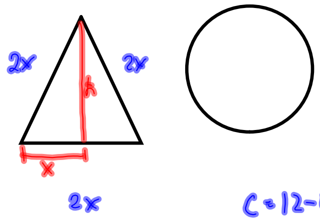
dim

$x = 8\sqrt{3}$

$y = \frac{240}{8\sqrt{3}} = 10\sqrt{3}$

2. A piece of wire 12 inches long is being used to make up to two figures: an equilateral triangle and a circle. How should the wire be divided so that the total area enclosed is a maximum? A minimum?





$h = x\sqrt{3}$

$C = 12 - 6x$

$2\pi r = 12 - 6x$

$\pi r = 6 - 3x$

$r = \frac{6-3x}{\pi}$

$$A = \frac{1}{2} b h + \pi r^2$$

$$A = \frac{1}{2}(2x)x\sqrt{3} + \pi\left(\frac{6-3x}{\pi}\right)^2$$

domain $[0, 2]$

$$A = x^2\sqrt{3} + \frac{(6-3x)^2}{\pi}$$

$$A = x^2\sqrt{3} + \frac{36}{\pi} - \frac{36x}{\pi} + \frac{9x^2}{\pi}$$

$$A' = 2x\sqrt{3} - \frac{36}{\pi} + \frac{18}{\pi}x$$

$$A'' = 2\sqrt{3} + \frac{18}{\pi}$$ >0 for any x. Thus any c.v. will be a min.

$$0 = 2x\sqrt{3} - \frac{36}{\pi} + \frac{18}{\pi}x$$

$$\frac{36}{\pi} = 2x\sqrt{3} + \frac{18}{\pi}x$$

$$36 = 2\pi x\sqrt{3} + 18x$$

$$36 = (2\pi\sqrt{3} + 18)x$$

$$x = \frac{36}{2\pi\sqrt{3} + 18} = 1.246$$ ← min

$A(0) = \frac{36}{\pi} = 11.46$

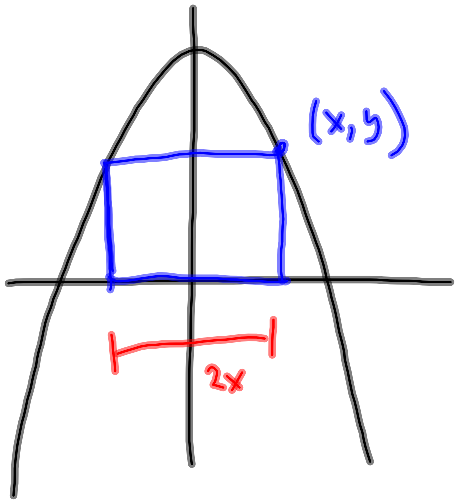
$A(2) = 4\sqrt{3} = 6.93$

max @ $x = 0$

max. no triangle all of the wire for a circle

Apr 17-2:54 PM

3. What are the dimensions of the largest rectangle that can be inscribed in the area bounded by the curve $\underline{y = 12 - x^2}$ and the x-axis?



$A = (2x)(y) = 2xy$

$A = 2x(12 - x^2)$

$A = 24x - 2x^3$

domain $[0, \sqrt{12}]$

$A' = 24 - 6x^2$

$A'' = -12x$

$A'' < 0$ for $0 < x \leq \sqrt{12}$

any c.v. will be a max.

<u>find c.v.</u>

$A' = 0$

$24 - 6x^2 = 0$

$24 = 6x^2$

$4 = x^2$

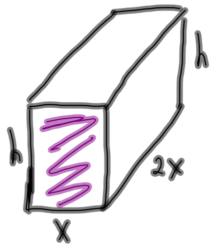
$x = 2$ c.v.

$y = 12 - 2^2 = 8$

dim. of the rectangle are 4 by 8

Apr 17-2:54 PM

4. A rectangular storrage container with an open top is to have a volumn of 10m^3. The length of the base is twice the width. Material for the base costs \$10 per square meter. Material for the sides cost \$6 per square meter. Find the cost of materials for the cheapest such container.



$V = x(2x)h = 2x^2h$

$10\,m^3 = 2x^2h$

$5 = x^2h \rightarrow \frac{5}{x^2} = h$

$\text{Cost} = 10(x \cdot 2x) + 6\left(2(xh) + 2(2x \cdot h)\right)$

$= 20x^2 + 36xh$

$C = 20x^2 + 36x\left(\frac{5}{x^2}\right)$

$C = 20x^2 + \frac{180}{x}$ domain $x > 0$

$C' = 40x - \frac{180}{x^2}$

find c.v.

$C' = 0$

$0 = 40x - \frac{180}{x^2}$

$\frac{180}{x^2} = 40x$

$180 = 40x^3$

$\frac{180}{40} = x^3$

$x^3 = \frac{9}{2} \rightarrow x = \sqrt[3]{4.5}$

$C'' = 40 + \frac{360}{x^3}$

for $x > 0$ $C'' > 0$

any c.v. will be a min.

$C(\sqrt[3]{4.5}) = \$163.54$

$y' = x^n$ if $n \neq -1$ $\quad y = \dfrac{x^{n+1}}{n+1}$

$y' = x^{-1} = \dfrac{1}{x}$ $\quad y = \ln|x|$

$y' = e^{Kx}$ $\quad y = \dfrac{1}{K} e^{Kx}$

Apr 17-6:18 PM

5. Find the most general antiderivative.

(a) $f'(x) = x^4 + \frac{8}{x} + \frac{3}{x^2} + \sqrt[3]{x} + 7$ $= x^4 + \frac{8}{x} + 3x^{-2} + x^{\frac{1}{3}} + 7x^0$

$$f(x) = \frac{x^5}{5} + 8\ln|x| + \frac{3x^{-1}}{-1} + \frac{x^{4/3}}{4/3} + 7x + C$$

(b) $f'(x) = \frac{10x^2 + 1}{2x^3}$ $= \frac{10x^2}{2x^3} + \frac{1}{2x^3} = \frac{5}{x} + \frac{1}{2}x^{-3}$

$$f(x) = 5\ln|x| + \frac{1}{2}\frac{x^{-2}}{-2} + C$$

$$= 5\ln|x| - \frac{1}{4x^2} + C$$

(c) $f'(x) = x^4(x^2 + 5)$ $= x^6 + 5x^4$

$f(x) = \frac{x^7}{7} + \frac{5x^5}{5} + C = \frac{x^7}{7} + x^5 + C$

(d) $f'(x) = \sec^2(x) + \sec(x)\tan(x)$

$f(x) = \tan(x) + \sec(x) + C$

(e) $f'(x) = 7e^x + \sqrt[5]{x^2}$ $= 7e^x + x^{2/5}$

$f(x) = 7e^x + \dfrac{x^{7/5}}{7/5} + C = 7e^x + \frac{5}{7}x^{7/5} + C$

(f) $f'(x) = \dfrac{4}{1+x^2} + 12e^{3x}$ $= 4 \cdot \dfrac{1}{1+x^2} + 12e^{3x}$

$f(x) = 4\arctan(x) + 12 \cdot \frac{1}{3}e^{3x} + C$

$f(x) = 4\arctan(x) + 4e^{3x} + C$

6. Find the position function of a partical whose movement can be described with the following information.

$\mathbf{a}(t) = \langle 3\sin(t), 2e^t \rangle$, $\mathbf{v}(0) = \langle 6, 3 \rangle$, $\mathbf{s}(\pi) = \langle 0, \pi \rangle$

$$V = \langle -3\cos(t) + c_1, 2e^t + c_2 \rangle$$

$$= \langle -3\cos(t), 2e^t \rangle + C \qquad \text{with } C = \langle c_1, c_2 \rangle$$

$$v(0) = \langle 6, 3 \rangle = \langle -3, 2 \rangle + C$$

$$\langle 6, 3 \rangle - \langle -3, 2 \rangle = C$$

$$\langle 9, 1 \rangle = C$$

$$V = \langle -3\cos(t) + 9, 2e^t + 1 \rangle$$

$$S = \langle -3\sin(t) + 9t, 2e^t + t \rangle + B$$

$$S(\pi) = \langle 0, \pi \rangle$$

$$\langle 0, \pi \rangle = \langle 9\pi, 2e^\pi + \pi \rangle + B$$

$$\langle 0, \pi \rangle - \langle 9\pi, 2e^\pi + \pi \rangle = B$$

$$B = \langle -9\pi, -2e^\pi \rangle$$

$$\boxed{S(t) = \langle -3\sin(t) + 9t - 9\pi, 2e^t + t - 2e^\pi \rangle}$$

Apr 17-2:56 PM

7. Find $f(x)$ if $f''(x) = 60x^3 + 6$ and $f(1) = 12$ and $f(-1) = 6$

$f'(x) = \frac{60x^4}{4} + 6x + C$

$f'(x) = 15x^4 + 6x + C$

$f(x) = \frac{15x^5}{5} + \frac{6x^2}{2} + Cx + B$

$f(x) = 3x^5 + 3x^2 + Cx + B$

$\underline{f(1) = 12}$

$12 = 3 + 3 + C + B$

$12 = 6 + C + B$

$6 = C + B$

$\underline{f(-1) = 6}$

$6 = -3 + 3 - C + B$

$6 = -C + B$

$6 + C = B$

$6 = C + (6 + C)$

$6 = 2C + 6$

$0 = 2C$

$C = 0$

$B = 6$

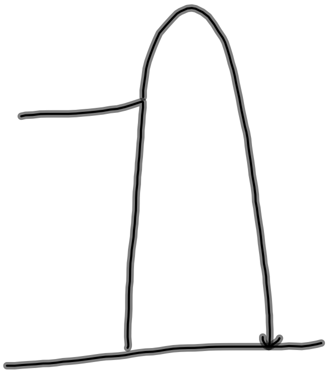
$f(x) = 3x^5 + 3x^2 + 6$

Apr 17-2:56 PM

8. A stone is thrown upward from a building 510 meters tall with a speed of 8 meters per second. Note: acceleration due to gravity is $9.8m/s^2$ or $32ft/s^2$

(a) Find the distance of the stone above the ground at time t.

(b) With what velocity does the stone hit the ground?



$s(0) = 510$ m

$v(0) = 8$ m/s

$a(t) = -9.8$

$v = -9.8t + C$

$8 = 0 + C$

$C = 8$

$v(t) = -9.8t + 8$

$s(t) = -4.9t^2 + 8t + B$

$510 = 0 + 0 + B$

$510 = B$

$\boxed{s(t) = -4.9t^2 + 8t + 510}$

$s(t) = 0$

$-4.9t^2 + 8t + 510 = 0$

by quadratic formula.

$t = -9.418$ or $t = 11.05$

$\boxed{v(11.05) = -100.29 \text{ m/sec}}$

9. A car is breaking with a constant deceleration of $60 ft/s^2$ producing skid marks measuring 240ft before coming to a stop. How fast was the car traveling when the breaks were first applied?

$a = -60$

$V = -60x + C$

$S = -30x^2 + Cx + B$

$0 = 0 + 0 + B$

$0 = B$

$S(0) = 0$

$S(\overset{N}{\text{stopped}}) = 240$

find $v(0)$ ie find C

$V(\overset{N}{\text{stopped}}) = 0$

$240 = -30n^2 + Cn$

$0 = -60N + C$

$60N = C$

$240 = -30n^2 + (60n)n$

$240 = -30n^2 + 60n^2$

$240 = 30n^2$

$8 = n^2$

$N = \sqrt{8} = 2\sqrt{2}$

$C = 60(2\sqrt{2})$

$= 120\sqrt{2}$ ft/sec