Spring 2012 Math 151

Sample Problems for Exam 3

sections: 4.2–4.6, 4.8, 5.1–5.3, 5.5, and 5.7

courtesy: Joe Kahlig

This collection of questions is intended to give an idea of different types of question that might be asked on the exam. This is not intended to represent an actual exam.

1. Find $f(x)$.

   (a) $f'(x) = x\sqrt[3]{x^2} + \dfrac{7}{x^2} + 3$

   (b) $f'(x) = \dfrac{x-8}{4x^2}$

   (c) $f'(x) = (x^2 - 3)^2$

   (d) $f'(x) = \sec(x)\tan(x) + 2sec^2(x)$

2. Find the function $f(x)$ if $f''(x) = 8\sin(x) + e^{2x}$ if $f(0) = 6$ and $f'(0) = 5$

3. Find $f'(x)$.

   (a) $y = \log_3(x^2 + 1) + 5^{x^3}$

   (b) $y = \arctan(5x) + \ln\sqrt{x^4 + 1}$

(c) $y = \sin^{-1}(3x^2)$

(d) $y = \left(\dfrac{x^2 + 2}{x^4 + 1}\right)^{5x}$

4. Find the value of A so that $c = 2$ will satisfy the conclusion of the Mean Value Theorem on the interval $[0, A]$ for $f(x) = x^3 + x - 1$.

5. Find the inverse function for $y = \dfrac{1 + 2^x}{3 - 2^x}$

6. If $f(x) = 4 + x + e^{3x-6}$ is one to one and $g(x)$ is the inverse of $f(x)$, then compute $g'(7)$.

7. Compute the exact value.

   (a) $\arccos\left(\cos\left(\frac{17\pi}{12}\right)\right) =$

   (b) $\arcsin\left(\sin\left(\frac{8\pi}{5}\right)\right) =$

   (c) $\cos(\tan^{-1}(5)) =$

8. Compute the exact values of these limits.

   (a) $\displaystyle\lim_{x\to 1^+}\left(\frac{x}{x-1} - \frac{1}{\ln(x)}\right)$

   (b) $\displaystyle\lim_{x\to 0}\left(\cos(3x)\right)^{-1/x^2}$

9. A population has the characteristic that at any given time(in years) its growth rate is three times the size of the population. At the end of the first year the size of the population is 70. Find a formula that gives the population where x is the number of years since the population started.

10. Solve for x. $\log x + \log(x + 6) = \log(2x - 4)$

11. Here is the **graph of the derivative f′(x)** of a function $f(x)$. Assume that the function $f(x)$ is continuous for all real numbers. Use the graph to answer the following questions.

    (a) Give the critical values of $f(x)$ and then tell which are local maximums, local minimums, or neither.
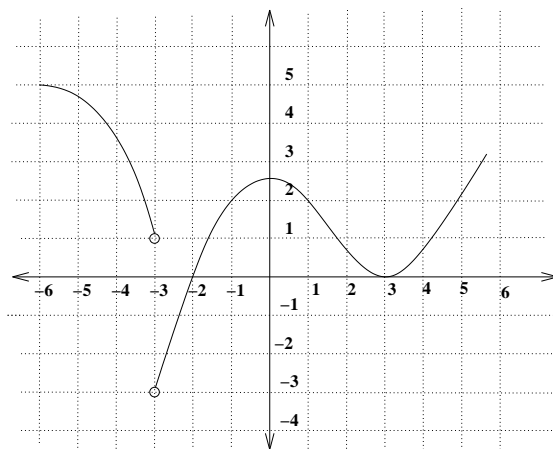
    (b) Find the intervals where $f(x)$ is decreasing and the intervals where it is increasing.

    (c) Find the intervals where $f(x)$ is concave down and the intervals where it is concave up.

    (d) Sketch a graph of $f(x)$.

12. Use the function $f(x) = \dfrac{1}{x(x-3)^2}$ to answer the following questions.

   (a) Find the intervals where the function is increasing and the intervals where it is decreasing.

   (b) Find the relative extreme of $f(x)$.

   (c) Sketch a graph of $f(x)$.

13. The domain of $f(x)$ is all real numbers. Find the intervals where $f(x)$ is concave up and where it is concave down. Give the x-values of the inflection points.

   $f''(x) = (x-9)^2(x-2)e^{(2x^3-6)}$

14. Find the absolute extreme values for the function on the given interval.

   (a) $f(x) = x^3 - 12x + 12$ on $[-3, 1]$

   (b) $g(x) = x^{-2}$ on $[-2, 1]$

15. Find the values of a and b so that $f(x) = ax^2 + b\ln(x)$ will have an inflection point at $(e^2, 5)$.

16. Find the point(s) on the graph of $y = \frac{x^2}{2}$ that is the closest to the point $(0, 19)$.

17. A wire 9 m long is to be cut into at most 2 pieces. The first piece, x, is bent into a square; the second piece is bent into a rectangle whose length is twice the width. How much of the wire should be used for the square so that the total area enclosed is a maximum? If this is not possible, then explain why.