Spring 2012 Math 151

Sample Problems for Exam 2
sections: Chapter 3, and 4.1
courtesy: Joe Kahlig

This collection of questions is intended to give an idea of different types of question that might be asked on the exam. This is not intended to represent an actual exam.

1. Compute these limits.

   (a) $\lim_{x\to\infty} \left(\frac{\pi^2}{16}\right)^{-x} =$

   (b) $\lim_{x\to-\infty} \left(\frac{\pi^2}{16}\right)^{-x} =$

   (c) $\lim_{x\to\infty} \frac{-10}{5+3e^{2x}} =$

   (d) $\lim_{x\to-\infty} \frac{-10}{5+3e^{2x}} =$

   (e) $\lim_{x\to\infty} \frac{3e^x+4}{7e^x+5} =$

2. Find the value of A for which this limit will have a non-zero answer. Also give the value of the limit.
   $\lim_{x\to-3} \frac{2\sin(x+3)}{x^2-x-A}$

3. Given $y = 4 - x^2$

   (a) Find Δy if x changes from $x = 1$ to $x = 1.5$

   (b) Find dy for $x = 1$ and $dx = 0.5$

4. Use differentials to approximate:

   (a) $\sqrt[4]{16.4}$

   (b) $\cos 28^o$

5. The circumference of a sphere is measured as 4m with an error of ± 0.2m. Use differentials to estimate the maximum error of the surface area of the sphere. What is the relative error and percent error?

6. Find the linear approximation for $y= e^{3(x-1)}$ at $x = 1$ and use it to approximate e^3.

7. Find $\dfrac{dy}{dx}$ for the following.

   (a) $y = e^{x^3} - e^2$

   (b) $y = \tan(5x)e^{4x+4}$

   (c) $y = \dfrac{2}{3x^3} + \sqrt[5]{x^2 + 7x}$

(d) $y = \dfrac{x^2 + 5}{3x - 2}$

(e) $y = \sec(5x^2)$

(f) $y = f(g(\sin(4x)))$

(g) $y^5 + x^2y^3 = 5\cos(3x)$

8. Find the equation of the tangent line to $y = \sec(x) - 2\cos(x)$ at $x = \pi/3$

9. Find the value(s) of x where the tangent line is horizontal for the function $f(x) = (x^2 + 2)^3(x + 4)^4$

10. Find the value(s) of A so that the function $f(x) = xe^{Ax^2}$ will have a horizontal tangent line at $x = 3$.

11. For what value(s) of c and m that will make the function $f(x)$ be differentiable everywhere. If this can not be done, then explain why. Fully justify your answers.

$$f(x) = \begin{cases} x^2 & \text{for } x < 3 \\ cx + m & \text{for } x \geq 3 \end{cases}$$

12. The curve is defined by

$x = 2t^3 - 3t^2 - 12t$
$y = t^2 - t + 1$

(a) Find all the values of t for which the tangent line is horizontal.

(b) Find all the values of t for which the tangent line is vertical.

(c) Find $\dfrac{dy}{dx}$ evaluated at the point $(-13, 1)$. Also give the equation of the tangent line at this point.

13. The position of a particle at time t is given by $\mathbf{r}(t) = \langle 8t^3 - 2t, 2t \rangle$.

(a) Find the velocity and the acceleration vector at $t = 1$

(b) Sketch the graph of the curve. Include in the picture the position and tangent vector at $t = 1$.

14. A particle moves on a vertical line so that its coordinate at time t is $y = t^3 - 12t + 3$, $t \geq 0$.

    (a) When is the particle moving upwards?

    (b) Find the distance that the particle travels in the time interval $0 \leq t \leq 3$.

15. A particle moves along the curve $y = \sqrt{1 + x^3}$. As it reaches the point $(2, 3)$, the y-coordinate is increasing at a rate of 4cm/s. How fast (in cm/s) is the x-coordinate of the point changing at that instant?

16. Two French sea horses frolic in the sea at Mediterranean Downs, practicing for the big race on Saturday. Phillip travels due north of the starting post, while Gigi heads Due east. At time t, Phillip's distance (in cm) from the post is $y = 7 + 2t + 0.5t^2$, whereas that of Gigi is $x = 6 + 4t$. Here time is measured in seconds. At what rate is the distance between Phillip and Gigi changing after $t = 2$ seconds?

17. A trough is 10 feet long and its ends have the shape of isosceles triangles that are 4 feet across the top and have a height of 1 foot. If the height of the water is decreasing at a rate of 0.15 ft/min when the water is 6 inches deep, find the rate of change of the volume of the tank at this time.