Fall 2007 Math 151
Final Exam Practice - Answers *courtesy:*
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Final Exam Practice: Sections 1.1 - 6.5

1. a.) $< -5, -7 >$

   b.) $< -1/\sqrt{10}, -3/\sqrt{10} >$

   c.) $153°$

   d.) Vector projection: $< 2/5, 6/5 >$;

   scalar projection: $\dfrac{-4}{\sqrt{10}}$

2. A vector equation: $< 1 + 2t, -2 + 10t >$;

   parametric equations: $x = 1 + 2t$, $y = -2 + 10t$

3. above force=25.88 N, below force=36.60 N

4. Formula to use: $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

   a.) $f'(x) = \dfrac{1}{2\sqrt{1+x}}$

   b.) $f'(x) = \dfrac{-1}{(x-3)^2}$

5. a.) ∞

   b.) $\dfrac{1}{4}$

   c.) $-\dfrac{1}{4}$

   d.) The limit does not exist because $\lim\limits_{x \to 3^+} f(x) = 17$
   and $\lim\limits_{x \to 3^-} f(x) = 5$

   e.) -3

   f.) $-\dfrac{1}{2}$

6. a.) 3

   b.) -2

   c.) -5

   d.) The limit does not exist

   e.) Not continuous at $x = 3$ (not in domain), not continuous at $x = -1$, $x = 5$ and $x = 7$ (the limit does not exist). Not differentiable at $x = -1$, $x = 3$, $x = 5$ and $x = 7$ (not continuous implies not differentiable). Also not differentiable at $x = -4$ and $x = -6$ because of sharp corners.

7. a.) Not continuous at $x = 0$ because $\lim\limits_{x \to 0} f(x)$ does not exist. Continuous for all other values of x.

   b.) Not continuous at $x = 1$ because $\lim\limits_{x \to 1} f(x) = 1$, yet $f(1) = 4$.

   c.) $a = 6$, $b = -3$

8. $x + y + 1 = 0$, $11x - y = 25$

9. horizontal asymptote: $y = 0$, vertical asymptote: $x = 1$.

10. $m = -8$, equation: $y = -8x$

11. 4

12. $x > \ln 4$

13. $f^{-1}(x) = \ln \dfrac{x}{1-x}$

14. a.) $y' = \dfrac{1 - 3x^2 y + 9x^2}{x^3 + 4y^3 - 1}$

   b.) $y' = \dfrac{\sin(x-y) + 2y - 4}{\sin(x-y) - 2x}$

15. a.) $f'(x) = \dfrac{12x^2 - 4x^4 - 16x}{(1-x^2)^2}$

   b.) $f'(t) = 3t^2 \cos(1 - t^2) + 2t^4 \sin(1 - t^2)$

   c.) $G'(x) = 12 \tan^2(4x - 1) \sec^2(4x - 1)$

16. 56

17. $y - \ln 27 = (3 + \ln 27)(x - \ln 3)$

18. At $t = -1$: $y + 17 = 6(x + 1)$; horizontal tangent: $y = 64$; verical tangent: $x = 0$

19. $-\dfrac{7}{4}$

20. $L(x) = \ln 2 + 1/2(x - 2)$,

   $Q(x) = \ln 2 + 1/2(x - 2) - 1/8(x - 2)^2$. The linear and quadratic approximations are useful in approximating the function for x sufficiently close to a.

21. $\dfrac{dh}{dt} = \dfrac{49}{36\pi}$ cm/min

22. 5/3 feet per second

23. 0.5 feet per second

24. $x = 2$

25. $x = \dfrac{2e}{1-e}$ Since this is not in the domain, there is no solution.

26. $t = 2/5$

27. $x = t, y = e + 2et$

28. $f'(x) = -\tan x$

29. $y' = y\left(\dfrac{\ln(1 + x + 3x^3)}{2\sqrt{x}} + \dfrac{\sqrt{x}(1 + 9x^2)}{1 + x + 3x^3}\right)$, where

$y = (1 + x + 3x^3)^{\sqrt{x}}$

30. 7.284 minutes

31. $t = .944$ hours

32. Inc: $(3, \infty)$, Dec: $(-\infty, 3)$, Local Min: $(3, -33)$; Local Max: None; Concave up: $(-\infty, 0)$ and $(2, \infty)$, concave down: $(0, 2)$, points of inflection: $(0, -6)$ and $(2, -22)$

33. $(0, \infty)$

34. Absolute Max: -1; Absolute min: -5

35. critical values: $x = -1$, $x = 1$, $x = 5$, f inc: $(-1, 1)$, $(5, \infty)$, ; f dec: $(-\infty, -1)$, $(1, 5)$; local min: $x = -1$, $x = 5$; local max: $x = 1$; f cu: $(-\infty, 0)$ and $(5, \infty)$; f cd: $(0, 4)$; inflection points: $x = 0$, $x = 5$.

36. a.) e^3

b.) 0

c.) 0

37. $2/5x^{5/2} - 4/3x^{3/2} + 2x^{1/2}$

38. 1.448745691

39. $5/\sqrt{29}$

40. $\dfrac{\pi}{3}$

41. $y' = -\dfrac{3}{\sqrt{1 - 9t^2}} - \dfrac{1}{\sqrt{t}(1 + t)}$

42. $2\sqrt{30}$ by $\dfrac{90}{\sqrt{30}}$

43. 5 feet by 10 feet long

44. $2x\sqrt{1 - x^8}$

45. $-\dfrac{5}{9(3x^3 - 1)} + C$

46. $\dfrac{e^{-3} - e^{-5}}{6}$

47. -2

48. $\ln|t^2 - 1| + C$

49. $\dfrac{1}{\pi}$

50. $140/3 - 4\ln 4$

51. $-1/3\ln|\cos 3x| + $C

52. $2.5\ln 5$

53. $\dfrac{1}{4\cos^4 x} + C$

54. $2/3(x + 1)^{1.5} - 2\sqrt{x + 1} + C$