Week in Review–Additional Chapter 2 Material

Section 2.2 and 2.3: Systems of linear equations.

- Gauss-Jordan row operations.
- Row-Reduced form (Reduced Row Echelon Form)
  - The first non zero number in a row is a one. Called a leading one.
  - The leading one is the only non-zero entry in its column.
  - The leading ones are positioned in a diagonal-like manner starting at the upper left going to the lower right.
- Types of solutions
  - no solutions when a row of the matrix gives a non-true statement, i.e. $0 = 5$.
  - exact solution (when in row-reduced form)
    - number of leading ones = number of variables
  - infinite solution (when in row-reduced form)
    - number of leading ones < number of variables

1. Which of these matrices are in row-reduced form (reduced row echelon form)?

   (a) $\left[\begin{array}{ccc|c} 1 & 0 & 3 & 2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{array}\right]$ (b) $\left[\begin{array}{ccc|c} 1 & 0 & 4 & 2 \\ 0 & 1 & 5 & 9 \\ 0 & 0 & 0 & 0 \end{array}\right]$

   (c) $\left[\begin{array}{cccc|c} 1 & 2 & 0 & 2 & 7 \\ 0 & 0 & 1 & 4 & 3 \end{array}\right]$ (d) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 0 \end{array}\right]$

2. Give the solutions to the system of equations represented by the augmented matrices.

   (a) $\left[\begin{array}{ccc|c} x & y & z & \\ 1 & 0 & 0 & 21 \\ 0 & 1 & 0 & 9 \\ 0 & 0 & 0 & 4 \end{array}\right]$ (b) $\left[\begin{array}{ccc|c} x & y & z & \\ 2 & 0 & 0 & 18 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 5 & 30 \end{array}\right]$ (c) $\left[\begin{array}{ccc|c} x & y & z & \\ 1 & 0 & 4 & 2 \\ 0 & 1 & 5 & 9 \\ 0 & 0 & 0 & 0 \end{array}\right]$

   (d) $\left[\begin{array}{cccc|c} x & y & z & w & \\ 1 & 2 & 0 & 2 & 7 \\ 0 & 0 & 1 & 4 & 3 \end{array}\right]$ (e) $\left[\begin{array}{ccc|c} x & y & z & \\ 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 0 \end{array}\right]$

3. Solve these system of equations with the Gauss-Jordan Method.

   (a) $x + y - 2z = -2$
   $y + 5z = 32$
   $3x + 4y + z = 34$

   (b) $2x - 4y + 6z = 20$
   $3x - 6y + z = 22$
   $-2x + 5y - 2z = -18$

4. solve these systems of equations by any method.

   (a) $3x + y = 9$
   $x + z = 4 + y$
   $z - 11 = -3x$
   $4x + 6y + 2z = 15 + 7y$

   (b) $x + 3y + z = 10$
   $2x + 7y - z = 21$
   $4x + 13y + z = 41$

   (c) $3x + 2y + 5z = 7$
   $x + 4y + z = 13$
   $4x - 5y + 2z = -9$
   $5x + 10y + 7z = 32$

5. Bob has budgeted \$840 to expand his video library with 60 DVDs. His local video store prices their DVDs at \$10 for an old release, \$16 for a semi-new release, and \$22 for a current release. How many DVDs from each category will Bob be able to buy assuming that he uses all of the budgeted money?

   (a) Set up and solve the word problem. If the solution is infinite, then place restrictions on the parameter.

   (b) How many different solutions are there for this word problem?