Week in Review # 6
Sections 3.1, 3.2, 3.3, 3.4, 3.5

1. Find the derivatives for the following functions. Assume that a, b, and k are constants.

(a) $y = \frac{3}{t^2} - 4t^6 + 5 = 3t^{-2} - 4t^6 + 5$

$\frac{dy}{dt} = y' = -6t^{-3} - 24t^5 = -\frac{6}{t^3} - 24t^5$

(b) $u = 3aq^4 + q^{1.45} + b^3$

$u' = 12aq^3 + 1.45q^{.45}$

(c) $f(x) = 6\sqrt[3]{x^4} + \frac{k}{2x^5} = 6x^{4/3} + \frac{k}{2}x^{-5}$

$f'(x) = 6 \cdot \frac{4}{3}x^{1/3} + \frac{-5k}{2}x^{-6} = 8x^{1/3} - \frac{5k}{2}x^{-6}$

(d) $y = \frac{5x^4 + 7x^2 - 2x + 3}{x}$

quotient Rule $\rightarrow$ $y' = \frac{x(20x^3 + 14x - 2) - (5x^4 + 7x^2 - 2x + 3)(1)}{x^2}$

$y = \frac{5x^4}{x} + \frac{7x^2}{x} - \frac{2x}{x} + \frac{3}{x} = 5x^3 + 7x - 2 + 3x^{-1}$

$y' = 15x^2 + 7 - 3x^{-2}$

Title: Mar 5-11:45 AM (1 of 11)

2. Find the equation of the tangent line for the function $y = x^3 e^{(x^2-4)} + 3x$ at $x = 2$

need point & slope.

point: $x=2$ means $y = 2^3 e^{(2^2-4)} + 3(2)$

$y = 8e^0 + 6$

$y = 14$

slope: $m_{tan} = y'(2)$

use product Rule.

$y = f \cdot g$

$y' = f'g + fg'$

$y = e^{f(x)}$

$y' = f'(x) e^{f(x)}$

$y' = \underbrace{3x^2}_{f'} \underbrace{e^{x^2-4}}_{g} + \underbrace{x^3}_{f} \cdot \underbrace{2x e^{x^2-4}}_{g'} + 3$

$m_{tan} = y'(2) = \left.\frac{dy}{dx}\right|_{x=2} =$

$y'(2) = 3(2)^2 e^{2^2-4} + 2^3 \cdot 2(2) e^{2^2-4} + 3$

$= 12e^0 + 32e^0 + 3$

$= 47$

answer) $\boxed{y - 14 = 47(x-2)}$

3. Find the value for a for the function $y = 4x^3 + ax^2 + 3x + 2$ so that the rate of change at $x = 1$ is 10.

$y'(1) = 10$

$y' = 12x^2 + 2ax + 3$

$10 = 12 + 2a + 3$

$10 = 15 + 2a$

$-5 = 2a$

$-\frac{5}{2} = a$

4. Find the values of x where the function $y = x^4 - 4x^2 + 10x + 5$ has an instantaneous rate of change of 10.

$$y' = 4x^3 - 8x + 10$$

$$10 = 4x^3 - 8x + 10$$

$$0 = 4x^3 - 8x$$

$$0 = 4x(x^2 - 2)$$

$4x = 0$ or $x^2 - 2 = 0$

$x = 0$ $\quad$ $x^2 = 2$

$x = \pm\sqrt{2}$

answer $x = 0, -\sqrt{2}, \sqrt{2}$

5. Suppose the depth of the water, in meters, is a function of time, in hours, since 6am is given by $y = 7 + 3.8\sin(0.628x)$. How quickly is the water rising or falling at 9am? at Noon?

$y' = 3.8(.628)\cos(.628x)$

$y' = 2.3864\cos(.628x)$

at 9Am $\quad y'(3) = -.73526\,m/hr$

at noon $\quad y'(6) = -1.9333\,m/hr$

6. If the position function for an object is given by $s(x) = 7x^3 - 15x^2 - x + 25$. Find the velocity function and the acceleration function.

position $s(t)$

velocity $v(t) = s'(t)$ $\quad v(t) = 21x^2 - 30x - 1$

acceleration $a(t) = v'(t) = s''(t)$ $\quad a(t) = 42x - 30$

7. Find the derivative for the following functions.

(a) $f(x) = \sqrt[6]{x^3 + 7x^2 + 6}$ $= (x^3 + 7x^2 + 6)^{1/6}$

$$f'(x) = \frac{1}{6}(x^3 + 7x^2 + 6)^{-5/6} \cdot (3x^2 + 14x)$$

(b) $h(t) = 3e^{3t^2+4} + 5^{4t-5}$

$$\boxed{\begin{aligned} y &= a^{f(x)} \\ y' &= f'(x)\, a^{f(x)} \ln(a) \end{aligned}}$$

$$h'(t) = 3 \cdot (6t)\, e^{3t^2+4} + 4\left(5^{4t-5}\right) \cdot \ln(5)$$

$$= 18t\, e^{3t^2+4} + 4 * 5^{4t-5} * \ln(5)$$

product Rule.

(c) $y = (x^3 + 5x)\cos(x^4)$

$$y' = (3x^2+5)\cos(x^4) + (x^3+5x)\cdot(-4x^3)\sin(x^4)$$

$$= (3x^2+5)\cos(x^4) - (x^3+5x)(4x^3)\sin(x^4)$$

(d) $y = 2^{\sin(3x)} + \ln(x^4 + 7x^3 + 15)$

$$y' = 3\cos(3x)\cdot 2^{\sin(3x)}\cdot\ln(2) + \frac{4x^3+21x^2}{x^4+7x^3+15}$$

(e) $g(x) = e^{-5x^2}\sqrt{x^8 + x - 6}$ $= e^{-5x^2}(x^8+x-6)^{1/2}$

$g'(x) = -10x\,e^{-5x^2}(x^8+x-6)^{1/2} + e^{-5x^2} \cdot \frac{1}{2}(x^8+x-6)^{-1/2} \cdot (8x^7+1)$

(f) $y = \dfrac{x^2 + 5}{x^4 - 2x - 1}$

$y = \dfrac{Hi}{Lo}$ $\quad y' = \dfrac{L\,dH - H\,dL}{L^2}$

$$y' = \frac{(x^4-2x-1)\cdot 2x - (x^2+5)(4x^3-2)}{(x^4-2x-1)^2}$$

(g) $y = \left(\dfrac{x^4 + 7x + 1}{\cos(5x)}\right)^4$

$$y' = 4\left(\frac{x^4+7x+1}{\cos(5x)}\right)^3 \cdot \left[\frac{\cos(5x)\cdot(4x^3+7) - (x^4+7x+1)\cdot(-5\sin(5x))}{(\cos(5x))^2}\right]$$

(h) $g(a) = \ln\left(2a + e^{\sin(3a)}\right)$

$$g'(a) = \frac{2 + 3\cdot\cos(3a)\,e^{\sin(3a)}}{2a + e^{\sin(3a)}}$$

(i) $y = \ln\left(\frac{7x^2+5}{10-x^5}\right) = \ln(7x^2+5) - \ln(10-x^5)$

$$y' = \frac{14x}{7x^2+5} - \frac{-5x^4}{10-x^5}$$

$$y' = \frac{14x}{7x^2+5} + \frac{5x^4}{10-x^5}$$

(j) $f(x) = \ln\left((x^4+5)^7 \cos(3x^2)\right)$

$$f(x) = \ln(x^4+5)^7 + \ln(\cos(3x^2))$$

$$= 7\ln(x^4+5) + \ln(\cos(3x^2))$$

$$f'(x) = 7 \cdot \frac{4x^3}{x^4+5} + \frac{-6x\sin(3x^2)}{\cos(3x^2)}$$

(k) $y = \overset{f}{7^{(2-3x^2)}} \overset{g}{\ln(5-2x)}$

$$y' = \underbrace{-6x\, 7^{2-3x^2} \cdot \ln(7)}_{f'} \cdot \underbrace{\ln(5-2x)}_{g} + \underbrace{7^{2-3x^2}}_{f} \cdot \underbrace{\frac{-2}{5-2x}}_{g'}$$

$$y = a^{f(x)}$$

$$y' = f'(x)\, a^{f(x)} \cdot \ln(a)$$