1. Fill in the blanks with the relationships between $f(x)$, $f'(x)$, and $f''(x)$.

$f'(x) > 0$ means that f(x) is inc.

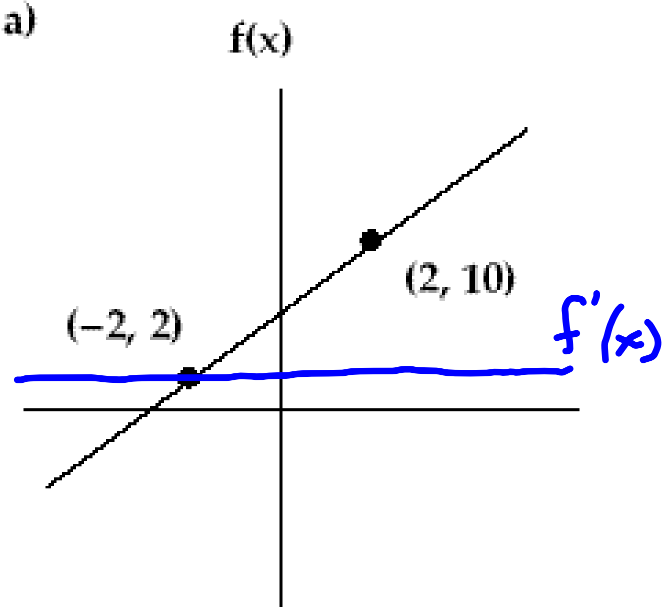
$f'(x) < 0$ means that f(x) is dec.

$f''(x) > 0$ means that f'(x) is inc. and f(x) is concave up

$f''(x) < 0$ means that f'(x) is dec. and f(x) is. concave down.

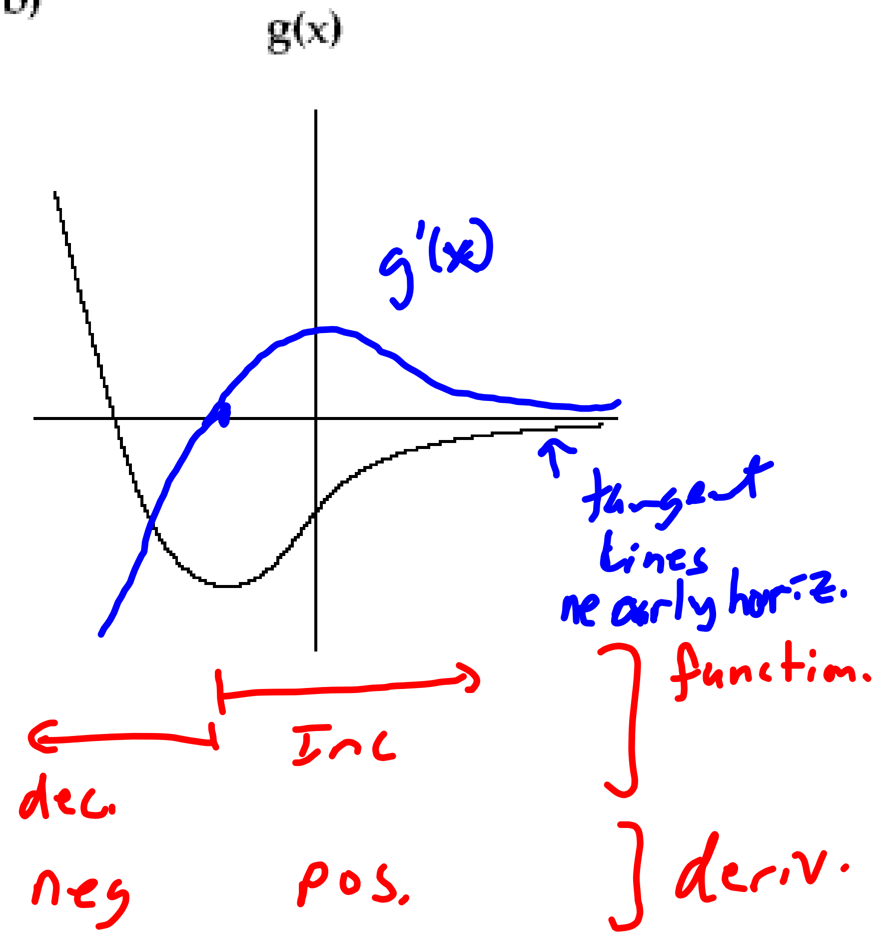
2. Sketch the graphs of the derivatives of each of these functions.

(a)

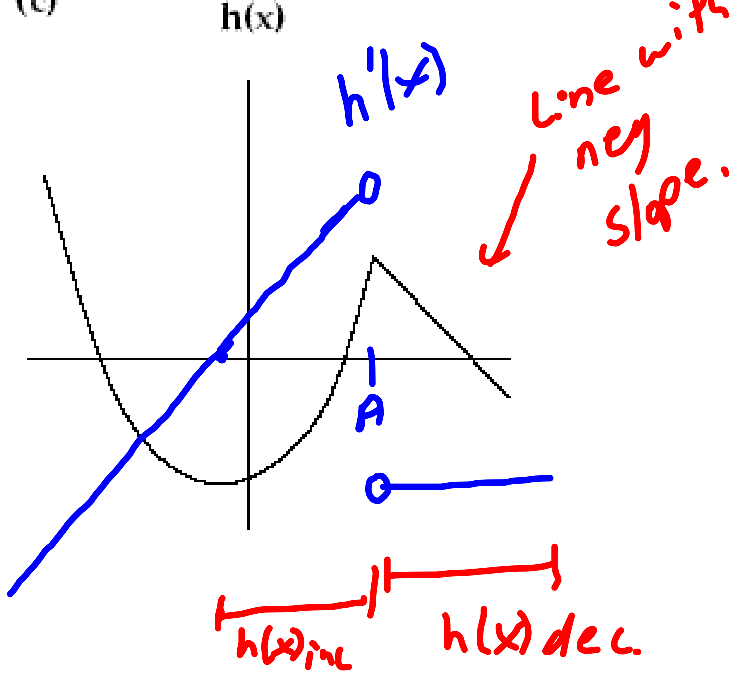


$m = \frac{8}{4} = 2$

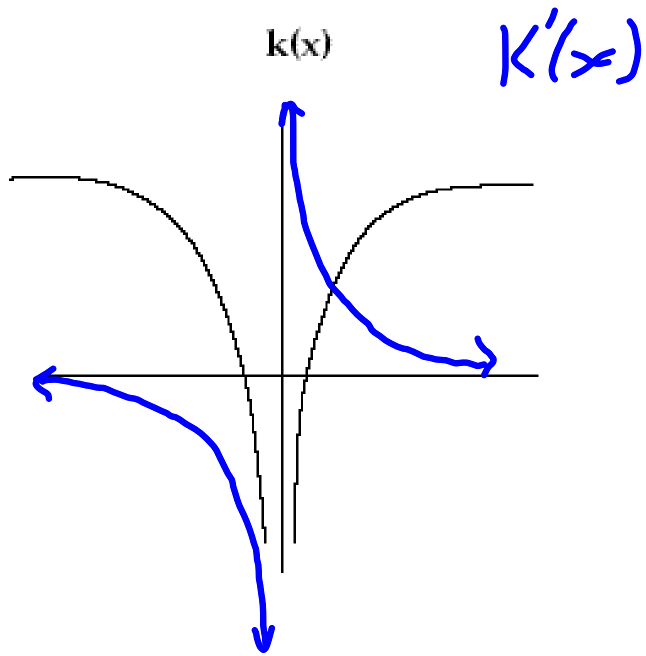
(b)



(c)



(d)



at x = A sharp point.

Note: A function doesn't have a deriv. at a sharp point.

3. Here is the graph of the function $f(x)$.

Look at tangent lines.

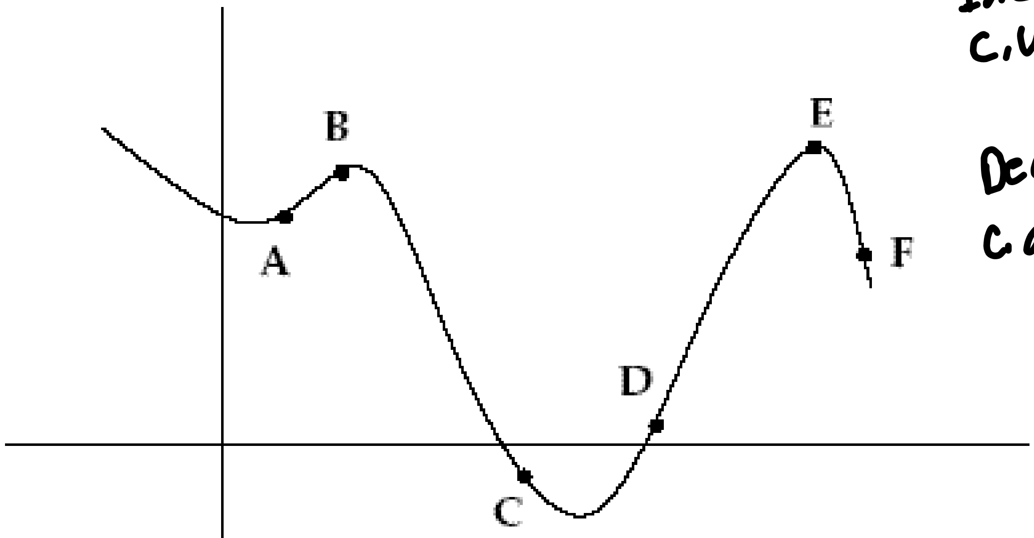
(a) Arrange the derivatives at the given points from smallest to largest.

smallest F, C, E, B, A, D largest

(b) At which points does $f'(x)$ and $f''(x)$ have the same sign?

Inc. c.u. pos: A, D

Dec c.d. neg: F

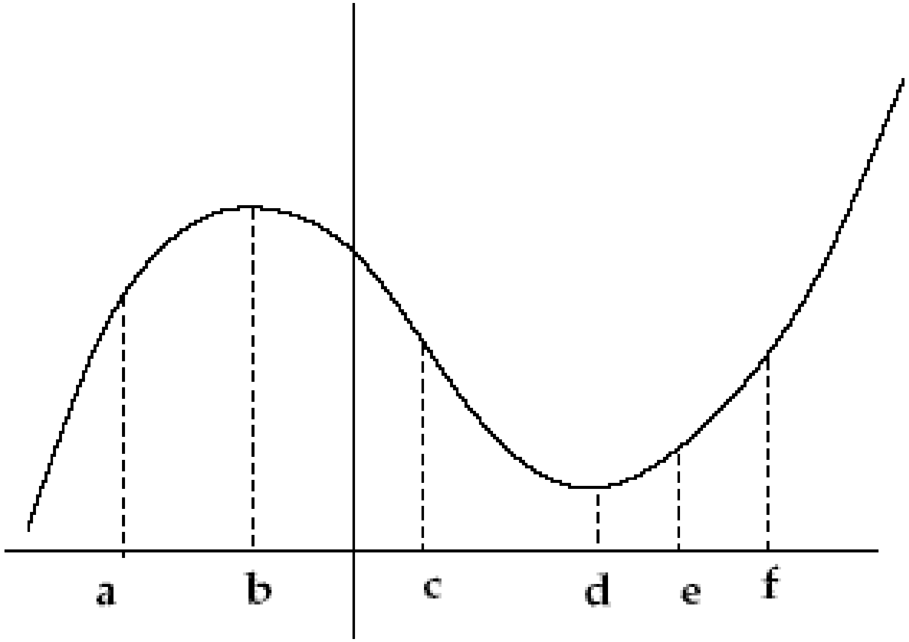


pos: D, A, B

neg: C, F

zero: E

4. Match the points with the derivatives.



x	d	e	B	c	a	f
$f'(x)$	0	1	0	-2	2	2
$f''(x)$	2	3	-2	0	-4	4

B/d

B/d.

e

f

time $\downarrow$ deg. $\downarrow$

5. Suppose $H = f(t)$ is the time, in minutes, that it takes a deep fryer to heat up to t^{0}F.

(a) What are the units of $f'(t)$ and what is the sign of $f'(t)$?

$$\frac{df}{dt} = \frac{\text{min}}{{}^{\circ}\text{F}}$$

pos.

(b) What is the meaning of $f(350) = 15$?

It takes 15min to heat to 350°F.

(c) what is the meaning of $f'(350) = 0.25$?

at the 350°F to go up 1°F it would take approx 0.25min.

(d) Estimate the time for the deep fryer to heat up to 375^{o}F.

$$\begin{array}{r} 375 \\ -\ 350 \\ \hline 25\ {}^{\circ}\text{F} \end{array}$$

$$f(375) \approx \underbrace{15}_{f(350)} + \underbrace{(0.25)}_{f'(350)}\underbrace{(25)}_{(375-350)}$$

$$\approx 21.25 \text{ min.}$$

yrs.

$P(t)$ is in \$

6. Suppose $P(t)$ is the monthly payment, in dollars, on a mortgage which will take t years to pay off.

(a) What are the units of $P'(t)$ and the sign of $p'(t)$?

$\frac{dP}{dt} = \$/\text{yr.}$ neg.

(b) What is the practical meaning of $P'(t)$?

how the monthly payments are decreasing for certain years.

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rate of change of the monthly payment as time increases.

7. Suppose $g(20) = 125$ and $g'(20) = -8$. Estimate $g(18)$, $g(25)$, and $g(31)$.

$$g(18) \approx \underbrace{125}_{g(20)} + \underbrace{(-8)}_{g'(20)} \underbrace{(-2)}_{(18-20)} = 125 + 16 = 141$$

$$g(25) \approx g(20) + g'(20)(25-20)$$
$$125 + (-8)(5) = 125 - 40 = 85$$

$$g(31) \approx \underbrace{125}_{g(20)} + \underbrace{(-8)}_{g'(20)} \underbrace{(11)}_{(31-20)} = 125 - 88$$
$$= 37$$

8. If $f(3) = 20$, $f'(3) = 2$ and $f''(x) < 0$ for $x \geq 3$, what can you say about the value of $f(7)$?

concave down.

find coord. of the point (7,28)

for tangent Line.

$y - y_1 = m(x - x_1)$

$y - 20 = 2(x - 3)$

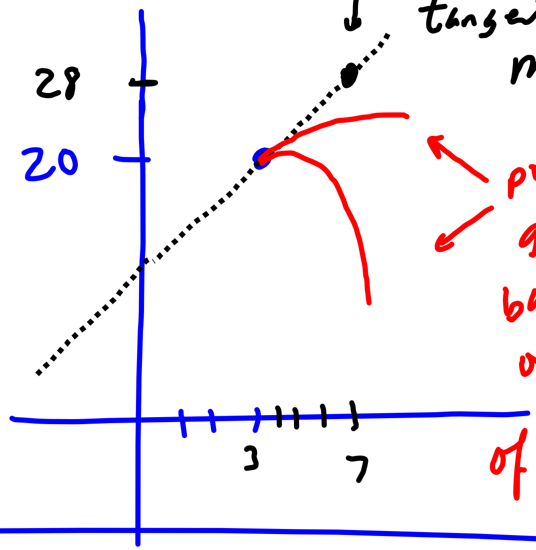
eq. of tangent Line

plug in a 7 for x

$y - 20 = 2(7 - 3)$

$y = 20 + 8$

$y = 28$



tangent Line
$m = 2$

possible graph based on the info. of $f''(x)$

If there was no concavity the function would go through the point (7, 28).

Since $f(x)$ is concave down for $x \geq 3$ we know $f(7) < 28$.

9. The temperature inside a house was given by $f(t)$ in °F. At 1pm, the temperature was 70°F. The first derivative, $f'(t)$ decreased until reaching a value of 1°F/hour at 1pm, then increased for the rest of the day. sketch a graph of the temperature inside the house during this time period.

unit of $f'(t)$ $\frac{°F}{hr}$ so units of t are hrs.

$f(x)$ is concave down	$f(x)$ concave up.
$f'(x)$ dec.	$f'(x)$ inc.

1pm

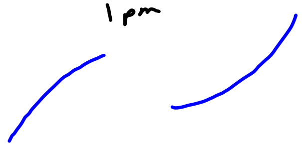
says that $f'(x) \geq 1\,°F/hr.$ all day long.

Meaning $f'(x)$ was pos. all day long.

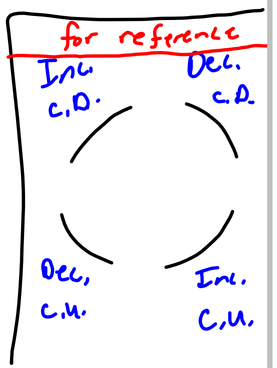
Says $f(x)$ is increasing all day long.

	c.d.	c.u.
$f(x)$	Inc.	Inc.

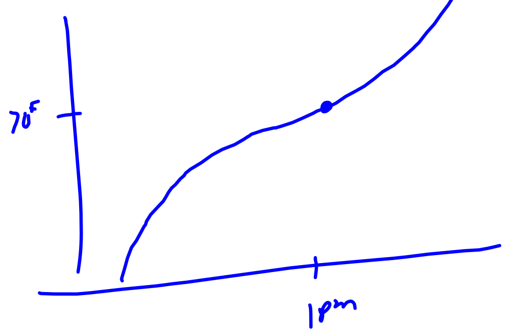
1pm



for reference



Sketch of $f(x)$



10. Sketch a graph of a function that meets these conditions.

$f(x)$ is positive for $x < 0$

$f'(x) > 0$ for $x < 3$

$f'(x) < 0$ for $x > 3$

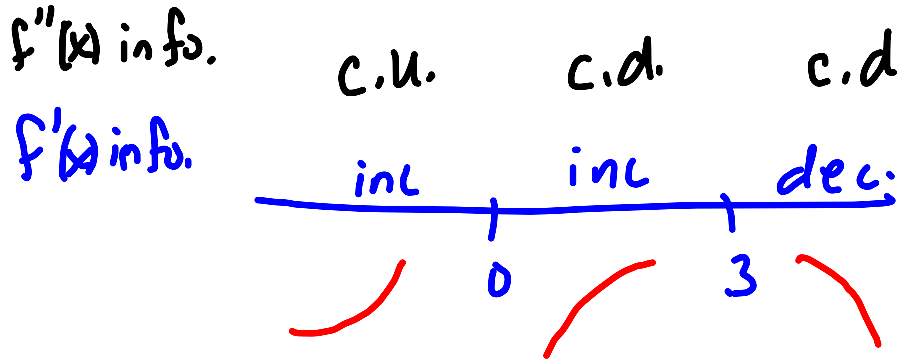
$f''(x) < 0$ for $x > 0$

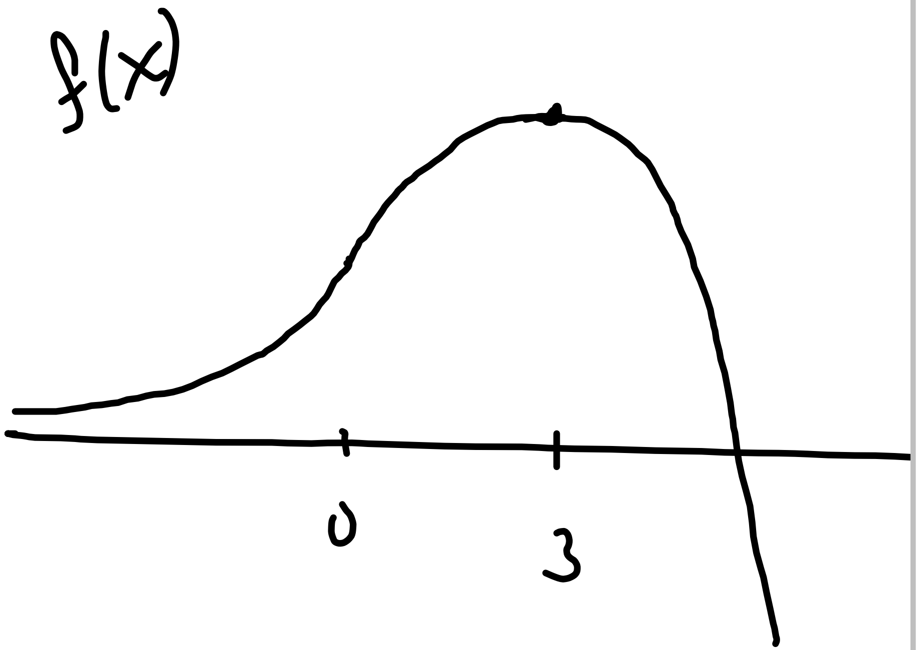
$f''(x) > 0$ for $x < 0$

$f'(3) = 0$

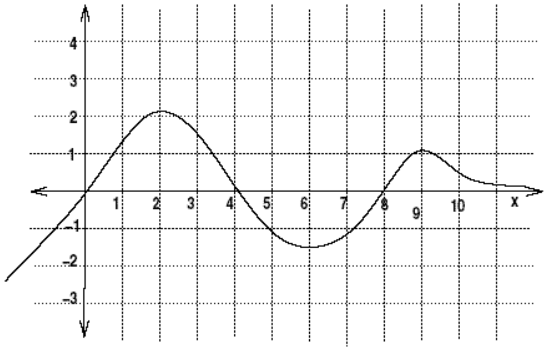
correction.

above x-axis for x<0





11. Here is the graph of $f'(x)$.



(a) On what intervals is $f(x)$ increasing? $0<x<4$, $x>8$

$f'(x)$ pos

(b) On what intervals is $f(x)$ decreasing? $x<0$, $4<x<8$

$f'(x)$ neg.

(c) On what intervals is $f(x)$ concave up? $x<2$, $6<x<9$

$f'(x)$ inc.

(d) On what intervals is $f(x)$ concave down? $2<x<6$, $x>9$

$f'(x)$ dec.

(e) Use the above information to sketch a graph of $f(x)$.

c.u./c.d. c.u. c.u. c.d. c.d. c.u c.u c.d.

Inc/dec dec Inc Inc dec dec Inc Inc.

0 2 4 6 8 9