Week in Review # 3
Section 1.6, 1.7, and 1.8

1. Simplify with logarithm rules.

(a) $\ln(x^4 z^5)$ $= \ln(x^4) + \ln(z^5)$

$= 4\ln(x) + 5\ln(z)$

(b) $\ln\left(\frac{e^{3x}}{x^5}\right)$ $= \ln(e^{3x}) - \ln(x^5)$

$= 3x - 5\ln(x)$

2. Solve for x.

(a) $J = 5 * 8^x$

$$\ln(J) = \ln(5 * 8^x)$$
$$\ln(J) = \ln(5) + \ln(8^x)$$
$$\ln(J) - \ln(5) = x\ln(8)$$
$$\boxed{x = \frac{\ln(J) - \ln(5)}{\ln(8)}}$$

Method # 2

$$\frac{J}{5} = 8^x$$
$$\ln\left(\frac{J}{5}\right) = \ln(8^x)$$
$$\ln\left(\frac{J}{5}\right) = x\ln(8)$$
$$\boxed{x = \frac{\ln(J/5)}{\ln(8)}}$$

(b) $8 * 3^x = 2 * 7^x$

$$\ln(8 * 3^x) = \ln(2 * 7^x)$$
$$\ln(8) + \ln(3^x) = \ln(2) + \ln(7^x)$$
$$\ln(8) + x\ln(3) = \ln(2) + x\ln(7)$$
$$\ln(8) - \ln(2) = x\ln(7) - x\ln(3)$$
$$\ln(8) - \ln(2) = x[\ln(7) - \ln(3)]$$
$$\boxed{\frac{(\ln(8) - \ln(2))}{(\ln(7) - \ln(3))} = x} = 1.6361$$

exact.

approx.

$a = 1 + r$

$P_o e^{kt}$

$\boxed{a = e^{k}}$

$\boxed{\ln a = k}$

3. For each of these formulas
   I) Convert them into the form $y = P_o a^t$ or $y = P_o e^{kt}$
   II) Give the relative rate of growth/decay.
   III) Give the continuous rate of growth/decay.

(a) $y = 35(1.25)^t$

I) $y = 35e^{.22314355t}$

II) growth 25%

III) growth 22.314355%

(b) $y = 27e^{-.127t}$

I) $y = 27(.88073367)^t$

II) decay 11.92663274%

III) decay 12.7%

4. When solving for the yearly continuous rate of decay for a substance a student rounded the answer to two decimal places and got the answer of 2%.

(a) If the correct rate of decay was 1.505%, find the half-life of the substance.

$$50 = 100e^{-.01505t}$$
$$\frac{1}{2} = e^{-.01505t}$$
$$\ln(.5) = -.01505t$$
$$\frac{\ln(.5)}{-.01505} = t \approx 46.05629 \text{ yrs}$$

(b) If the correct rate of decay was 2.4%, find the half-life of the substance.

$$50 = 100e^{-.024t}$$
$$\vdots$$
$$\frac{\ln(.5)}{-.024} = t \approx 28.8811 \text{ years}$$

(c) How does the half-life of the student's rate of decay compare with the answers in parts (a) and (b).

$$50 = 100e^{-.02t}$$
$$\vdots$$
$$\frac{\ln(.5)}{-.02} = t \approx 34.657359 \text{ years}$$

Student's answer has a very different half-life.

5. A bank account was started with \$600. Two years later the account had \$850.

(a) What is the continuous interest rate for the account?

$B = Pe^{rt}$

$$850 = 600e^{2r}$$

$$\frac{850}{600} = e^{2r}$$

$$\ln\left(\frac{850}{600}\right) = 2r$$

$$r = \frac{\ln\left(\frac{850}{600}\right)}{2}$$

$$r = .174153347$$

17.4153347%

(b) How long will it take for the account triple?

$$3 = 1e^{.174153347t}$$

$$\ln(3) = .174153347t$$

$$\frac{\ln(3)}{.174153347} = t \approx 6.3083 \text{ yrs}$$

6. You have been offered a payment of \$5000 in 4 years and a payment of \$8000 in 7 years. What is the present value of this offer if the interest rate is

(a) 6.25% compounded annually.

$$\boxed{B = P(1+r)^t \rightarrow P = B(1+r)^{-t}}$$

$$5000(1+.0625)^{-4} + 8000(1+.0625)^{-7}$$

\$9,156.77

(b) 6.25% compounded continuously.

$$\boxed{B = Pe^{rt} \rightarrow P = Be^{-rt}}$$

$$5000e^{-.0625(4)} + 8000e^{-.0625(7)}$$

\$9059.19

7. For the functions $f(x) = \sqrt{x+5}$ and $g(x) = 2x^2 + 3$ find

(a) $g(f(7)) = g(\sqrt{12}) = 2(\sqrt{12})^2 + 3$

$f(7) = \sqrt{12}$

$= 2(12) + 3$

$= \boxed{27}$

(b) $f(g(2)) = f(11) = \sqrt{16}$

$g(2) = 11$

$= \boxed{4}$

(c) $g(3 + g(1)) = g(3+5) = g(8)$

$g(1) = 5$

$= 2(8)^2 + 3$

$= \boxed{131}$

(d) $f(g(x)) = f(2x^2+3)$

$= \sqrt{(2x^2+3) + 5}$

$= \boxed{\sqrt{2x^2+8}}$

(e) $g(f(x)) = g(\sqrt{x+5}) = 2(\sqrt{x+5})^2 + 3$

$= 2(x+5) + 3$

$= 2x + 10 + 3$

$= 2x + 13$

8. Find two function $f(x)$ and $g(x)$ such that $h(x) = f(g(x))$

$h(x) = 5\ln(3x^2 + 1)$

more than one solution

$g(x) = 3x^2$

$f(x) = 5\ln(x+1)$

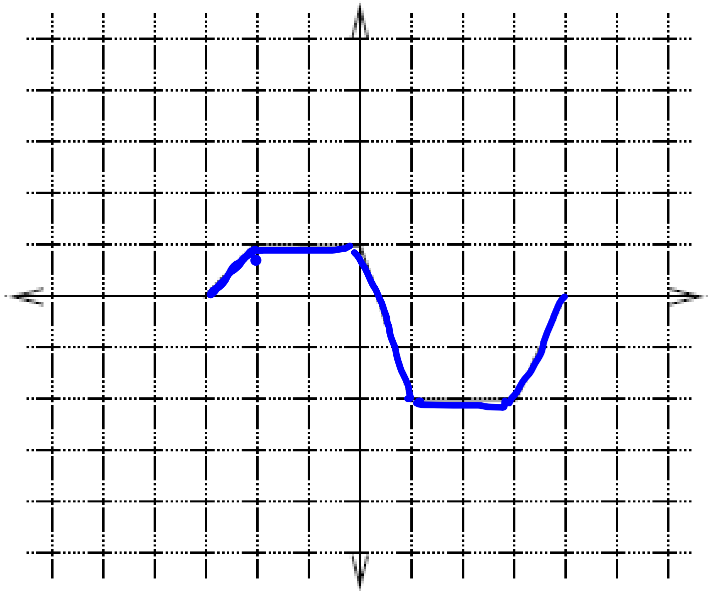
$f(g(x)) = f(3x^2)$

$g(x) = 3x^2 + 1$

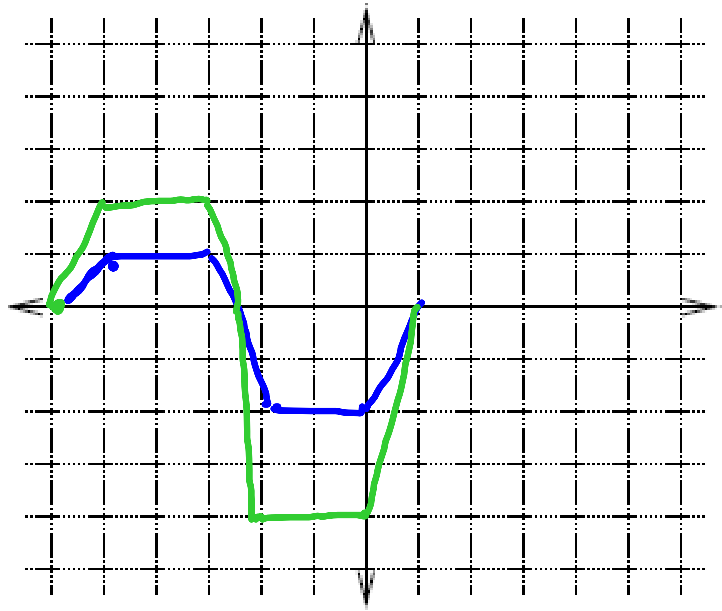
$f(x) = 5\ln(x)$

9. The graph of $f(x)$ is given. Use it to sketch the following.

$2f(x+3)$



(a) f(x+3)



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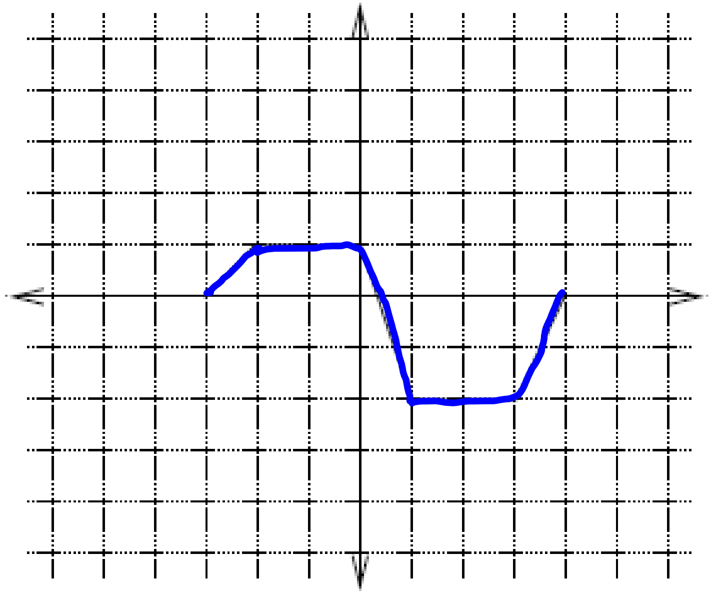
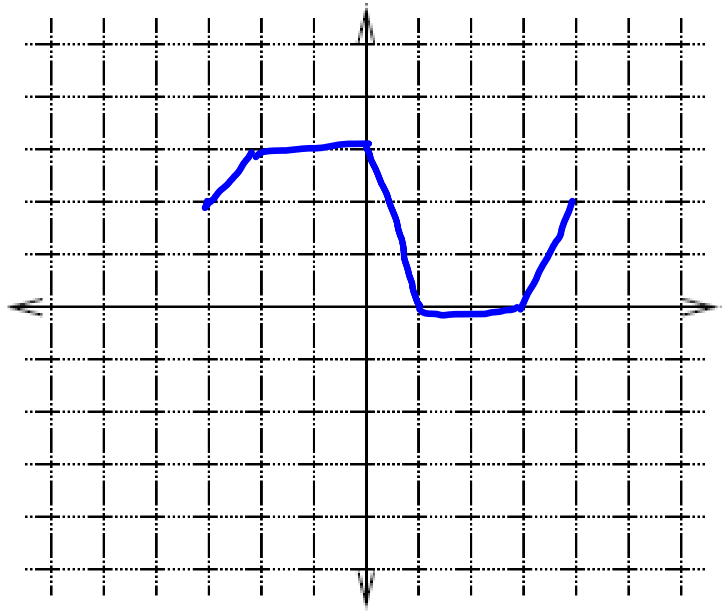
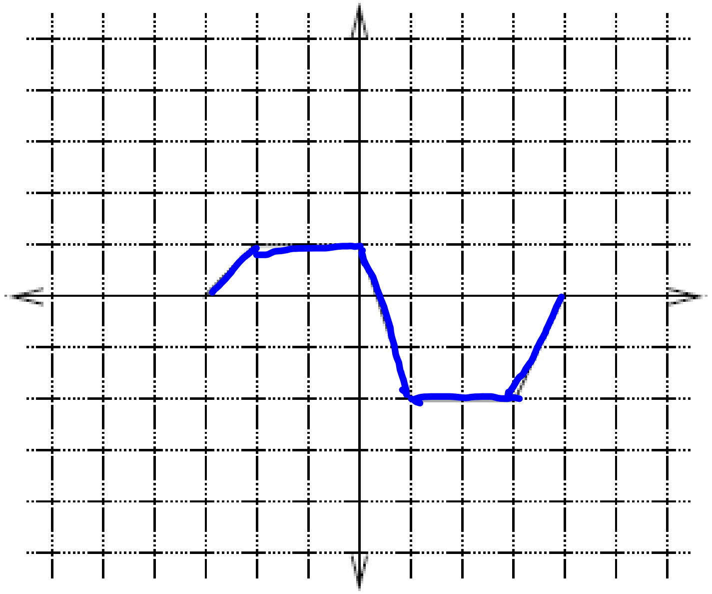


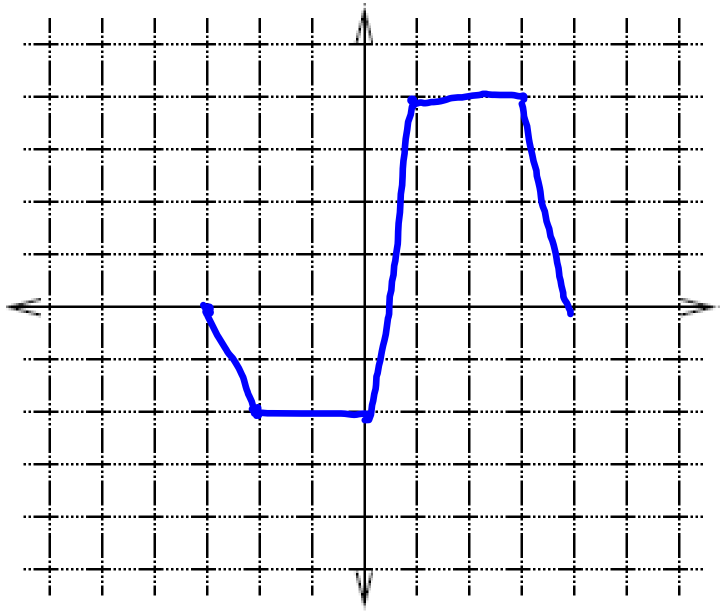
(b) f(x)+2



9. The graph of $f(x)$ is given. Use it to sketch the following.

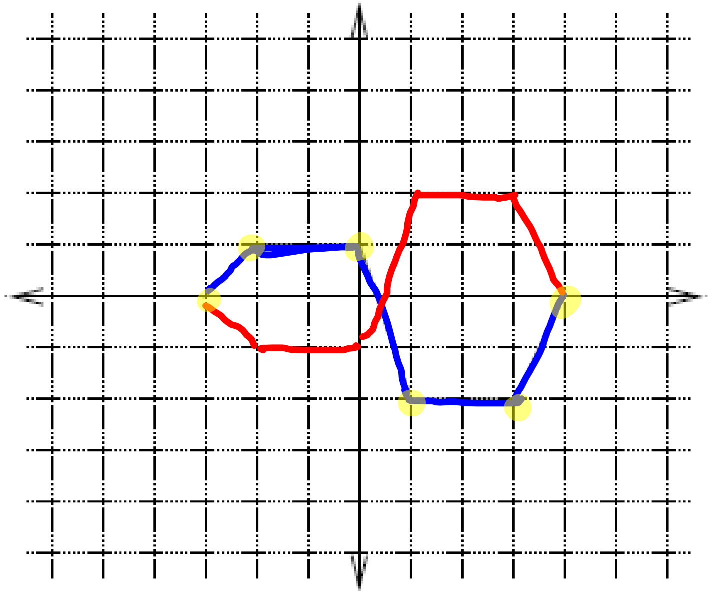


(c) $-2f(x)$



9. The graph of $f(x)$ is given. Use it to sketch the following.

$-f(x)$



(d) $2 - f(x) = -f(x) + 2$