Week in Review # 11
Sections 10.2(some), 10.3, 10.4

product rule.

1. Is $y = 5e^x + xe^x$ is a solution to the differential equation $y'' + 2x = 3y$?

$y' = 5e^x + 1e^x + xe^x$

$y' = 6e^x + xe^x$

$y'' = 6e^x + 1e^x + xe^x$

$= 7e^x + xe^x$

L.S.

$7e^x + xe^x + 2x$

R.S

$3(5e^x + xe^x)$

$15e^x + 3xe^x$

not equal, it is not a sol.

2. Find the values of n so that $y = 2x^n$ is a solution to $x^2y'' = 6y$

$y' = 2nx^{n-1}$

$y'' = 2n(n-1)x^{n-2}$

$$x^2\left(2n(n-1)x^{n-2}\right) = 12x^n$$

$$2n(n-1)\cdot x^2 \cdot x^{n-2} = 12x^n$$

$$2n(n-1)\,x^{2+n-2} = 12x^n$$

$$2n(n-1)x^n = 12x^n$$

These must be the same.

$2n(n-1) = 12$ → $n(n-1) = 6$

OR

$2n^2 - 2n = 12$

$2n^2 - 2n - 12 = 0$

$(2n-6)(n+2) = 0$

$2n = 6 \quad n = -2$

$n = 3$

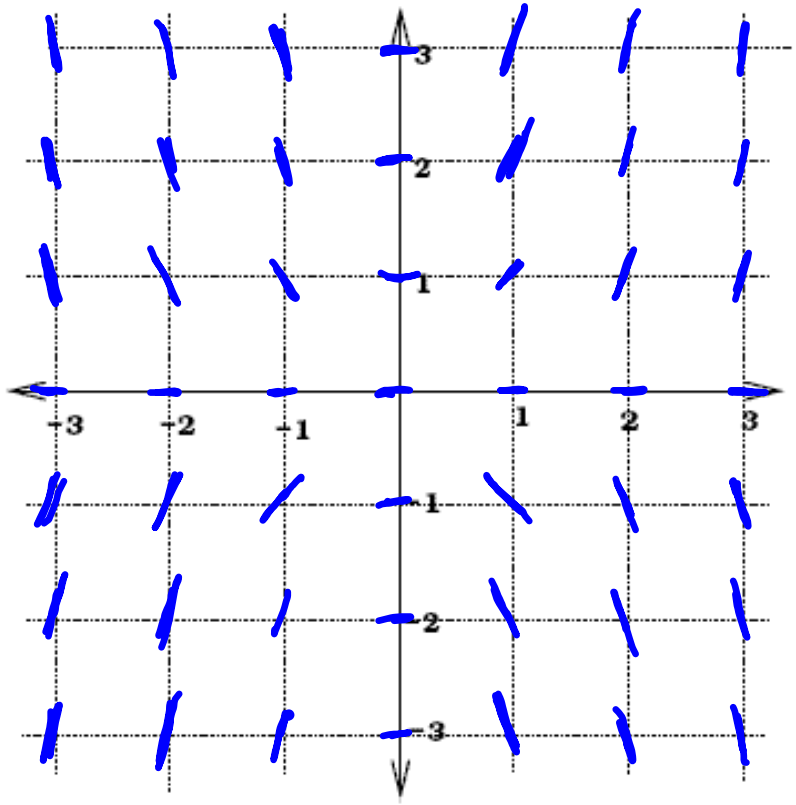
$n^2 - n - 6 = 0$

$(n-3)(n+2) = 0$

$n = 3 \quad n = -2$

Solution $n = 3$, $n = -2$

3. draw the slope field for $y' = xy$

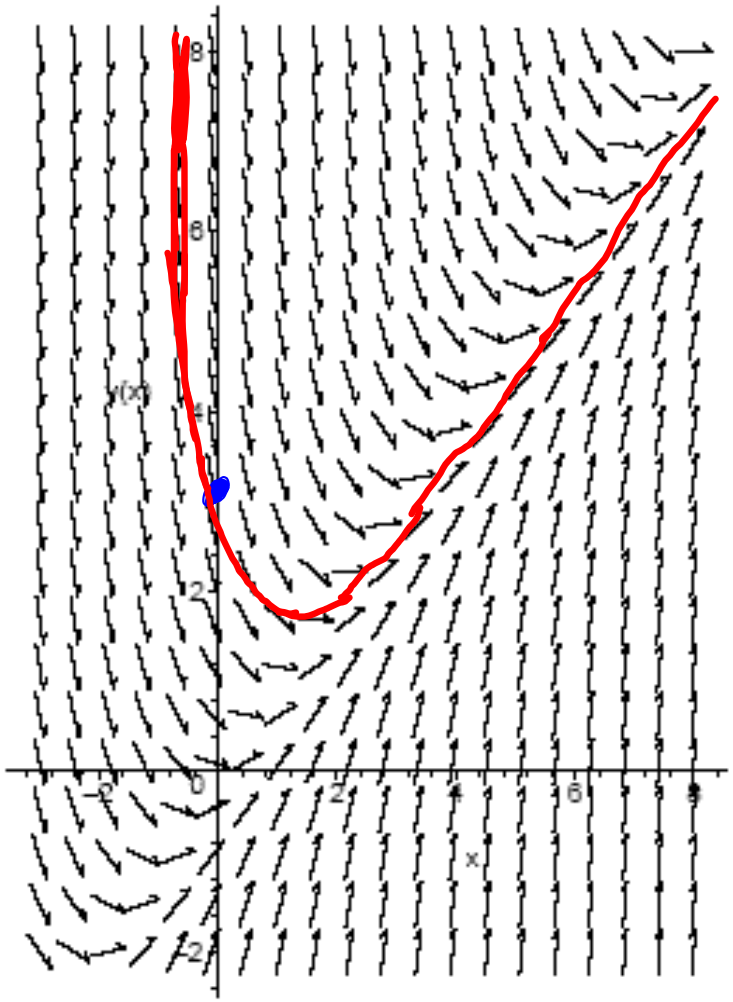


if x, y or both are 0 then $y' = 0$

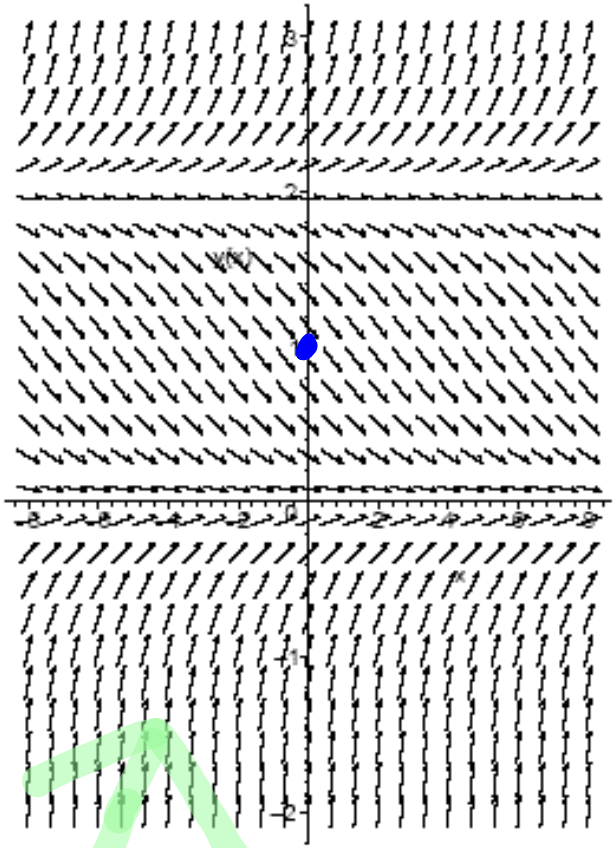
x	y	y'
1	1	1
1	2	2
1	3	3
1	-1	-1
1	-2	-2
1	-3	-3

x	y	y'
2	1	2
2	2	4
2	3	6
2	-1	-2
2	-2	-4
2	-3	-6

4. The slope field for the differential equation is given below. Draw the solution to the differential equation that has $y(0) = 3$



5. Match the slope field to the appropriate differential equation.



if $y=0$ or $y=2$ then $y'=0$

The only two choices that are possible are D and E

now test a pt.

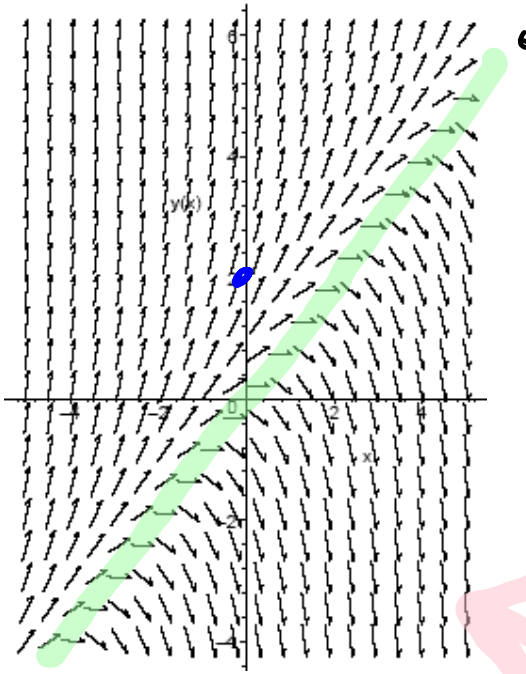
pick (0,1)

A) $y' = x + y$ B) $y' = 2 - y$ C) $y' = y - x$

D) $y' = 0.3(2 - y)y$ E) $y' = 0.3(y - 2)y$ F) $y' = x - y$

$+$ $-$

5. Match the slope field to the appropriate differential equation.



$y'=0$

D, E are not possible

B if $y=2$ then $y'=0$ ← not true

for A if $x=-y$ then $y'=0$ ↓ ie $(3,-3)$ $(4,-4)$ $(-2,2)$

A) $y' = x + y$ ~~X~~

B) $y' = 2 - y$ ~~X~~

C) $y' = y - x$ $2-0=2$ pos

D) $y' = 0.3(2 - y)y$ ~~X~~

E) $y' = 0.3(y - 2)y$ ~~X~~

F) $y' = x - y$ $0-2=-2$ neg

not points on green bar so not it

pick a test point. say $(0,2)$

by picture y' is pos. at $(0,2)$

Answer is C.

$y' = ky \rightarrow y = Ce^{kx}$

6. Solve these differential equations subject to the initial conditions.

(a) $\frac{dy}{dx} = -.5y$ and $y = 8.4$ when $x = 0$. ← use to solve for c.

$y' = -.5y$

Sol $y = Ce^{-.5x}$

$8.4 = Ce^{0}$

$8.4 = C$

$y = 8.4e^{-.5x}$

(b) $\frac{1}{y}\frac{dy}{dx} = 7$ and $y = 20$ when $x = 2$.

$\frac{1}{y}y' = 7$

$y' = 7y \rightarrow y = Ce^{7x}$

$20 = Ce^{7(2)}$

$20 = Ce^{14}$

$\frac{20}{e^{14}} = C = 1.663057 \times 10^{-5}$

$= .0000166 = .0000166$

$y = .0000166e^{7x}$

(c) $y' + 1.1y = 0$ and $y(1) = 3$

$y' = -1.1y$

$y = Ce^{-1.1x}$

$3 = Ce^{-1.1}$

$\frac{3}{e^{-1.1}} = C = 9.0125$

$y = 9.0125e^{-1.1x}$

(d) $5y' - 9y = 0$ and $y(0) = 20$

$5y' = 9y$

$C = 20$

$y' = 1.8y$

$y = Ce^{1.8x}$

$y = 20e^{1.8x}$

7. A cup of coffee contains about 250 mg of caffeine. Caffeine is metabolized and the rate that it leaves the body is proportional to the amount of caffeine in the body. After 6 hours there are 75mg in the body.

(a) Write a differential equation that measures the amount of caffeine, A, in the body as a function of the number of hours, x, since the coffee was consumed.

$A' = kA$

A = amt of caffeine

(b) Solve this differential equation.

points given

$(0, 250) \rightarrow C = 250$

$(6, 75)$

$A = Ce^{kx}$

$A = 250e^{kx}$

$75 = 250e^{6k}$

$\frac{75}{250} = e^{6k}$

$\ln\left(\frac{75}{250}\right) = 6k$

$\frac{1}{6}\ln\left(\frac{75}{250}\right) = k$

$k = -.2007$

$y = 250e^{-.2007x}$